Essential Sand Geotechnical Parameters for use in Advanced Soil Models

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This document is a compact reference note that brings together practical definitions, typical value ranges, and commonly used empirical correlations for sand, with frequent pointers to the original literature and a consistent reminder that correlations have limits and should only be applied within their intended scope. The content is organized into four parameter groups, each presented as its own table: (1) dilatancy and strength parameters including relative density, critical-state and peak friction, dilatancy measures, and CPT-based correlations intended for simpler strength and deformation descriptions; (2) critical-state and critical-state–based parameters, such as the critical-state void ratio e_c *and the state parameter* $\Psi = e - e_c$*, used to interpret whether a sand is likely contractive or dilative and to support more state-aware assessments; (3) small-strain stiffness parameters for the Hardening Soil Small (HSS) framework, cantered on* G_{max} *and strain-dependent stiffness reduction; and (4) intergranular strain overlay parameters used with hypoplastic models to improve small-strain and cyclic-response representation when full calibration data are not available. Taken together, the tables function as a practical parameter-selection cheat sheet that links common index and test inputs (e.g., CPT, triaxial results, grain-size descriptors) to parameters used in Mohr–Coulomb (MC), Hardening Soil (HS), Hardening Soil Small (HSS), and hypoplasticity (HP) models, while emphasizing careful, context-aware application and encouraging readers to consult the primary sources. A summary of the content of each table is given in the following*

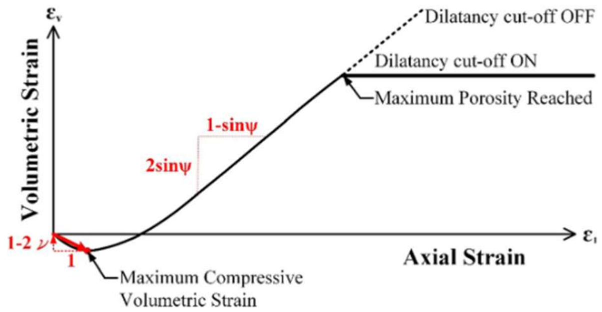
- Table 1: Dilatancy and strength parameters of sand
  - Defines relative density and compiles CPT-based correlations for estimating D_r from cone resistance (e.g., Schmertmann; Jamiolkowski et al.; Kulhawy & Mayne), noting important limitations (e.g., dependence on consolidation/stress level).
  - Summarizes ways to estimate maximum dilatancy angle (from drained triaxial data and Bolton-type correlations).
  - Provides guidance on critical-state friction angle and peak friction angle (including Bolton-style links between peak friction and dilatancy, and CPT-based estimation approaches such as Robertson).
  - Includes typical friction angle and cohesion ranges used in practice (e.g., handbook-style suggested values), with context on why an apparent cohesion may sometimes be used for sands in simplified Mohr–Coulomb fits.
- Table 2: Critical state and critical-state-based parameters
  - Presents formulations for critical state void ratio e_c versus mean effective stress p': classic semi-log linear form (Roscoe et al.) and commonly used nonlinear alternatives (e.g., Wang et al.; Bauer), including links to grain-size descriptors (e.g., c_u, d_{50}) in hypoplasticity-oriented calibrations.

- Defines the state parameter $\Psi = e - e_c$, explains how it's used to infer contractive vs. dilative tendencies, and notes why it is often more informative than relative density (while still not capturing everything—stress ratio and fabric effects are discussed).

- Table 3: Small-strain stiffness parameters (Hardening Soil Small, HSS)
  - Summarizes widely used structures for small-strain shear modulus G_{max}as a function of void ratio, effective stress level, and stress-history/memory (OCR, loading direction).
  - Gives common modulus-reduction-with-strain functional forms and practical "rule-of-thumb" suggestions for situations with limited test data (including how to relate $G_{\mathrm{max}}$to unload–reload stiffness and typical reference strain levels for sands).
- Table 4: Intergranular strain parameters (hypoplastic models)
  - Provides practical default suggestions for the intergranular strain overlay parameters (used to improve small-strain response and cyclic behavior in hypoplasticity), including guidance for R, amplification factors (e.g., m_R, m_T), and nonlinearity parameters (χ, β_r), plus a relationship tying these to a "memory" strain scale.

Table 1: Dilatancy and strength parameters of sand

Parameter	Suggested value(s)/correlation(s)	Models the parameter can be used in
Relative density* (D_r) *Relative density is one of the index parameters used to characterise sands. However, it should be applied with care when describing deformation behaviour, because it cannot uniquely classify sands across different levels of confining pressure.*	Definition: $$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} x100\ [\%], I_D = D_r/100$$ Schmertmann (1976) [1] $$D_r = \frac{1}{C_2} \ln\left(\frac{q_c}{C_0\ (\sigma'_{v0})^{C_1}}\right)$$ C_0, C_1 and C_2 are empirical correlation factors. For Several NC sands, Schmertmann found $C_0 = 0.05$, $C_1 = 0.7$ and $C_2 = 2.91$. This correlation has been put to the test, and some limitations were pointed out in the literature. (One of them being the correlation works for normally consolidated soils.) Jamiolkowski et al. 2001 based on Lancellotta (1983) [2] $$D_r = B \cdot \ln\left(\frac{q_c/p_a}{(\sigma'_{v0}/p_a)^{C_1}}\right) - A$$ A, B and C_1 are parameters. Jamiolkowski et al. suggested A =67.5, 82.5, and 82.5 for high medium and low compressibility sands respectively, they also suggested B =26.8 and $C_1 = 0.5$. Kulhawy and Mayne (1990) [3] $$D_r = \left(\frac{q_c/p_a}{305 \cdot Q_C \cdot Q_{OCR} \cdot (\sigma'_{v0}/p_a)^{0.5}}\right)^{0.5}$$ in which Q_C is compressibility factor (0.91 for high, 1.0 for medium and 1.09 for low) and Q_{OCR} is overconsolidation factor (=$OCR^{0.18}$) with suggested values respectively 2.3, 5.1 and 10.1 for the low, medium and high OCR data.	Can be used for estimating other parameters,

Maximum dilatancy angle (ψ_{max})	 Determination from drained triaxial tests: $$\psi_{max} = \sin^{-1}\left(-\frac{\Delta\varepsilon_v}{-\Delta\varepsilon_v + 2\Delta\varepsilon_a}\right)_{max}$$ Bolton 1986 [4] $$\left(-\frac{\Delta\varepsilon_v}{\Delta\varepsilon_a}\right)_{max} = 0.3\left[I_D\left(Q - \ln p'_f\right) - R\right]$$ $$I_D = D_r/100$$ Q and R are empirical fitting parameters. Bolton found Q=10 and R=-1 for quartz and feldspar. But several others found some other values for the respective fitting parameters than Bolton's (Chakraborty and Salgado 2010, Cinicioglu and Abadkon 2015.) In addition, the constancy of Q was also questioned by other researchers who found out that Q depends on the effective confining stress (Chakraborty and Salgado 2010.)	MC, HS, HSS
Critical state friction angle (φ_c)	For quartz sand: $$\varphi_c = 30 \text{ degrees}$$ Caquot [5] $$\varphi_c = \tan^{-1}(\pi \tan\varphi_\mu /2)$$ where φ_μ is the interparticle friction angle , and $\varphi_\mu = 20^0$ gives $\varphi_c \approx 29.8$ degrees Bishop [6] $$\varphi_c = 15\tan\varphi_\mu/(10 + 3\tan\varphi_\mu)$$ $\varphi_\mu = 20^0$ gives $\varphi_c \approx 26$ degrees	HP,
Peak friction angle (φ_p)	SVV HB 220 [7]: **36 degrees** for compacted sand back and in front of abutments and support walls, for compacted dense sand under foundations, for uncompacted natural dense sand under foundation footings	MC, HS, HSS,

	33 degrees for uncompacted natural sand back and in front of abutment and support wall, loose sand under foundation footings Been et al. (1987) [8] $$\varphi_p = 32(1 - 2\Psi_0)$$ Ψ_0= initial state parameter, the in-situ value of the state parameter may be obtained from CPT tests. Bolton (1986) [4] $$\varphi_p = \varphi_c + 0.8\psi_{max}$$ $$\varphi_p = \varphi_c + 0.3I_R, I_R = I_D(10 - lnp) - 1$$ Robertson (2010) [9]: $$\varphi_p = \varphi_c + 15.84 \log Q_{tn,cs} - 26.88$$ $$Q_{tn,cs} = K_c Q_{tn}$$ $$Q_{tn} = \left[\frac{q_t - \sigma_v}{p_a}\right]\left(\frac{p_a}{\sigma_v}\right)^n$$ K_c =correction factor to correct normalized cone resistance in silty sands to an equivalent clean sand value	
Cohesion (For sands mixed with clays, cohesion due to bonding may be expected, Cohesion due to bonding is not expected for clean sand but cohesion may be used to compensate for the part of strength due to interlocking and the reduction in friction due to the Mohr-Coulomb linearization of the failure envelope...)	SVV HB 220 [7]: **0-10 kPa** for compacted sand back and in front of bridge abutments and support walls, for compacted dense sand under foundation footings, for uncompacted natural dense sand under foundation footings **0 kPa** for uncompacted natural sand back and in front of bridge abutments and support wall, loose sand under foundation footings	MC, HS, HSS,

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Table 2: Critical state and critical state-based parameters

NB: All correlations must be used with caution. When one decides to use a particular correlation, it is a good practice that one consults the original publication and investigates the appropriate application area, the background data and the limitations of the correlation. The table here under is for information and bring upfront some references which can be useful for estimation of parameters related to critical state and critical state-based parameters for sands.

MC = Elastic perfectly plastic Mohr-Coulomb model, HS = Hardening Soil model, HSS = Hardening Soil Small, HP = hypoplasticity model

Parameter	Suggested value(s)/correlation(s)	Models the parameter can be used in
Critical state void ratio (e_c)	Roscoe et al. [1] $$e_c = e_{ca} - \lambda_c \ln \frac{p'}{p_a}$$ where e_{ca} and λ_c are model parameters, p' is the effective confining pressure and p_a is atmospheric pressure. This is linear in a semi-log ($e - \log p'$) plot. However, several data for sands show a non-linear trend in a semi-log plot. The following two equations are commonly used to describe how the critical-state void ratio of sand varies with effective confining pressure. Wang et al [2] $$e_c = e_{ca} - \lambda_c \left(\frac{p'}{p_a}\right)^{n_w}$$ Bauer [3] $$e_c = e_{c0} \exp\left\{-\left(\frac{3p'}{h_s}\right)^{n_B}\right\}$$ e_{c0}, e_{ca}, n_w, n_B and h_s are assumed material parameters (h_s is called granulate stiffness in hypoplasticity models for sands.) Both equations show the desired non-linear trend in the semi-log plot. Herle and Gudehus [4] proposed fuctions that relate the parameters n_B and h_s with coefficient of uniformity c_u and mean grain diameter d_{50} (mm) $$n_B = 0.366 - 0.0341 \left(\frac{c_u d_0}{d_{50}}\right)^{0.33}$$ $$h_s = 542.5\text{x}10^{2.525\left(\frac{d_{50}}{d_0\sqrt{c_u}}\right)^{0.33}}$$	HP, most recent models

	$d_0 = 1$ mm is taken as a reference grain size. Remark: There has been active investigation of the influence of fines content on the critical state void ratio. There are suggestions of using equivalent granular void ratio instead of the void ratio to account for the effect of fines content in the literature. The critical state line may be estimated from cone penetration tests [5]	
Critical state friction angle	See Table 1: **Dilatancy and strength parameters of sand** Note that the critical state friction for triaxial compression is considered as input in most models and the shear mode dependency of the critical state friction angle is considered via the lode angle dependency in yield criterion. (The assumption in the Mohr-Coulomb yield function is that the friction angle is shear mode independent, i.e., triaxial compression and triaxial extension and other intermediate modes have the same friction angle, while the Lade-Duncan and the Matsoka-Nakai criteria give a critical state friction (and any other friction angle at a given shear strain mobilization, the peak friction angle for instance) that depends on the intermediate stress state. Investigation on the shear mode dependency (Lode angle dependency) of the criterion angle for Hostun sand can be found in [6]	HP, most recent models
State parameter (Ψ)	Been and Jefferies 1985 [6] Definition $$\Psi = e - e_c$$ where e is the current void ratio, e_c is the critical state void ratio at the same effective confining pressure. The state parameter has proven valuable both in engineering practice and in the development of models for sandy soils. It has been correlated with dilation, undrained shear strength, and drained shear resistance, and it has been incorporated into several constitutive frameworks. As a true state variable, the state parameter evolves with deformation and is therefore not strictly fixed.	Peak friction angle and dilatancy angle may be estimated based on the insitu state parameter MC, HS, HSS

Nevertheless, the initial state parameter can be used to characterise deformation behaviour. Even its sign provides a useful qualitative indication of likely response: a positive initial state parameter generally suggests contractive behaviour, whereas a negative value suggests dilative behaviour.
There are several correlations in the literature that were proposed for estimating the *in-situ* state parameter from cone penetration tests and dynamic tests. Eg. [8] [9] [10] [5] and shear wave velocity tests [11] [12].

Note: The state parameter is generally a more informative indicator than relative density. Been and Jefferies introduced it to address key limitations of relative density, particularly the fact that relative density does not account for confining pressure, which can strongly influence deformation behaviour. Their original definition, however, did not incorporate the effect of shear stress (or stress ratio) on the response. As a result, two sand states with the same state parameter may still behave differently if they are associated with different shear stress ratios. Subsequent work has proposed modifications to include stress-ratio effects. Fabric effects are also important—something Been and Jefferies recognised when proposing the state parameter. Dafalias and co-workers have explored this further, although these considerations are typically more relevant for constitutive modelling than for straightforward characterisation of sand deposits.

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Table 3: Small strain stiffness parameters for HSS (mainly for Sand)

The importance of accounting for small-strain behaviour when modelling soil deformations is now widely recognised. Surprisingly, there is still no generally accepted definition of what constitutes “small” strain. Stiffness near the onset of deformation is relatively high, but it decreases sharply as strain increases.

NB: All correlations should be applied with caution. If you choose to use a particular correlation, it is good practice to consult the original publication and review its intended range of applicability, the underlying dataset, and any stated limitations. The table below is provided for information and highlights references that may be useful when estimating small-strain stiffness parameters for the Hardening Soil Small model.

MC = Elastic perfectly plastic Mohr-Coulomb model, HS = Hardening Soil model, HSS = Hardening Soil Small, HP = hypoplasticity model

Parameter	Suggested correlation(s)/ value(s)	Models the parameter can be used in
Maximum shear stiffness at small strains (G_{max}) (Both G_{max} and G_0 have been used in the literature for denoting the same. In drained cyclic loading one may achieve G_{max} which is higher than G_0.)	Several correlations may be put in the form [1] based on many previously proposed equations [2], [3] , [4], [5] and several others) $$G_{max} = \alpha_G f_e f_\sigma f_m p_a$$ where α_G is assumed a material parameter, f_e is the void ratio dependency function, f_σ is effective stress dependency function and f_m is memory dependency function, p_a is atmospheric pressure ($f_s = f_e f_\sigma$ may be called state dependency function [6]). Wichtmann et al [7], based on specially mixed grain size distribution curves of quartz sand, proposed: $$\alpha_G = 0.5[1563 + 3.13c_u^{2.98}][\exp(-0.3f_c^{0.85}) + \exp(-0.28f_c^{1.1})]$$ where c_u is the coefficient of uniformity and f_c is fines content. The usual form of the stress-dependency function is of the form, e.g., [8] $$f_\sigma = f_p = \left(\frac{p}{p_a}\right)^n$$ where p is the effective confining stress, n is stress dependency parameter that ranges between 0.5 and 1 for several types of soils. $n = 0.5$ may be used for sandy soils, and n =1 or close to 1 may be used for clay soils. Whichtman et al. [7], based on specially mixed grain size distribution curves of quartz sand, proposed $$n = 0.4c_u^{0.4}[1 + 0.116\ln(1 + f_c)]$$	HSS

	Hardin and Richart (1963) [2] proposed $$f_e = \frac{(a_e - e)^2}{1+e}$$ In general, a_e in the range 1.4 - 7.3 has been reported, *e.g.*, [2, 9, 10, 11]. Higher values are reported for clays. A possible form that may be considered account for memory dependency is the following multiplicative format where f_{OCR} accounts for the OCR dependency and f_θ accounts for the effect of change of loading direction [1]. $$f_m = f_{OCR} f_\theta$$ Hardin and Black (1968) [9] proposed $f_{OCR} = OCR^{n_o}$ (the effect of OCR may be taken into account through the void ratio dependedncy function, f_e.) f_θ is less relevant for cases that concern monotonic loading. But, this dependency is clearly documented in Atkinson et al. [3]. The dependency on loading direction (the current compared to previous loading directions) is linear with the change of direction at lower stress ratios and less at higher stress ratios. Several correlations and values of constants for the determination of the small strain stiffness can be found in tables 3.2, 3.3 and 3.4 on page 32 and 33 of Benz' s PhD thesis [5].	
Parameters for describing the decay of secant shear modulus with shear strain within the small strain range	A general form that is often used to represent this decay is [1], see also [6]: $$\frac{G_{sec}}{G_{max}} = \frac{1}{1 + a_\gamma \left(\frac{\gamma}{\gamma_{ref}}\right)^{\alpha_\gamma}}$$ where γ is the shear strain, γ_{ref} is a reference shear strain, and a_γ, α_γ and γ_{ref} are model parameters (Tsegaye, 2014). Early proposition by Hardin and Drnevich (1969) [8], for example, was $a = 1, \alpha_\gamma = 1$ and $\gamma_{ref} = \tau_{max}/G_{max}$, where τ_{max} is the maximum shear stress. Stokoe et al., (2004) [12] considered $a = 1$, $\alpha_\gamma = 1$ and $\gamma_{ref} = \gamma_{0.5}$. Santos and Correia (2001) [13] modified the same equation considering $a_\gamma = 0.385$, $\alpha_\gamma = 1$ and $\gamma_{ref} = \gamma_{0.7}$. The reference shear strains $\gamma_{0.5}$ and $\gamma_{0.7}$ are shear strains at which the stiffness has decayed by 50% and 30% respectively. In Benz (2007) [5], $\gamma_{0.7}$ is determined at a slightly different point, where the stiffness has degraded 27.8%. When a_γ and α_γ are fixed, only one parameter, namely γ_{ref} is required to define the strain dependency curve. A cut-off criterion is required for using the above equation as an overlay for enhancement of elastoplastic models. The cut-off criterion may be either in terms of stiffness or in terms of strain. Where there is lack of data, defining $\frac{\gamma_{cut-off}}{\gamma_{0.7}} = 10$, the	HSS for instance

cut-off stiffness for the small strain overlay may be set $\frac{G_{sec}}{G_{max}} = \approx 0.2$.

The tangent form of the above equation is [1]

$$\frac{G_{tan}}{G_{max}} = \frac{1 + a_\gamma \bar{\gamma}^{\alpha_\gamma}(1 - \alpha_\gamma)}{\left(1 + a_\gamma \bar{\gamma}^{\alpha_\gamma}\right)^2}, \bar{\gamma} = \frac{\gamma}{\gamma_{ref}}$$

which for Benz's [5] implementation in the Hardening Soil Small (HSS) model simplifies to:

$$\frac{G_{tan}}{G_{max}} = \frac{1}{\left(1 + 0.385 \frac{\gamma}{\gamma_{0.7}}\right)^2}.$$

Considering $\frac{G_{sec}}{G_{max}} \approx 0.2$, for the HSS model, if one does not have better data, one may use the following for rough estimate of G_{max}^{ref} based on the reference unload-reload reference (E_{ur}^{ref})

$$G_{max}^{ref} = 5G_{sec,cut-off} = \frac{E_{ur}^{ref}}{2(1 + v_{ur})}$$

where E_{ur}^{ref} is the reference unload-reload stiffness and v_{ur} is unload-reload Poison's ratio. In general, v_{ur} may vary from close to zero to 0.3. A default value of $v_{ur} = 0.2$ is considered in the HS model in Plaxis. Where this specific choice is reasonable, one may use

$$G_{max}^{ref} = 5\frac{E_{ur}^{ref}}{2.4} \approx 2.08E_{ur}^{ref}$$

G_{max}^{ref} twice E_{ur}^{ref} can be easily remembered.

At reference pressure, say $p_{ref} = p_a$, the reference shear strain

$$\gamma_{0.7}^{ref} = 10^{-4}$$

may be considered for sands. $\gamma_{0.7}$ at other effective confining pressure levels may be estimated by [5, 14]

$$\gamma_{0.7} = \gamma_{0.7}^{ref} \left(\frac{p}{p_a}\right)^{\hat{n}}$$

For clays, $\gamma_{0.7}$ depends on plasticity index and generally increases with increasing plasticity index. The plots by Vucetic and Dobry (1991) [15] may be used for a first estimate where data is lacking. Refer to [16] for correlations for clay soils in general and Norwegian Clays in particular.

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Table 4: Some suggestions for the estimation of Intergranular Strain parameters for Hypoplastic models (Sand) in the absence of data

Hypoplasticity was developed as an alternative framework for soil modelling. However, the relatively soft response at the onset of deformation and the unrealistic ratcheting observed during cyclic loading in early hypoplastic models motivated the introduction of a small-strain overlay known as the Intergranular Strain concept (Niemunis and Herle, 1997). This overlay extends the hypoplastic formulation by adding five additional parameters.

Note: All correlations should be used with caution. When selecting a correlation, it is good practice to consult the original publication and assess its intended range of applicability, underlying data, and limitations. The table below is provided for information and highlights references that may be useful for estimating Intergranular Strain parameters for hypoplastic models tailored to sand deformation behaviour.

Parameter	**Suggested correlation(s)/ value(s)**	**Models the parameter can be used in**
R	Describes the “elastic range” of the Intergranular Strain. For sands, this parameter may be set to a default value of $R = \gamma_{0.7}$. where $\gamma_{0.7}$ is the shear strain level at 3=% degradation of the small strain stiffness. In the absence of proper test for the calibration of the parameter, $R = 10^{-4}$ may be used as suggested by [35].	Extended hypoplasticity models (for Sands)
m_R	m_R is a parameter that amplifies the hypoplastic stiffness to account for the high tangent stiffness at the initiation of deformation and during loading reversal. Considering the Santos and Correia [32] stiffness degradation rule, and assuming the swept out of memory strain at the shear strain level 10^{-3} and $R = 10^{-4}$ for sands, one obtains $$m_R \approx 5$$ so the same as the default value suggested by Niemunis and Herle [35].	
m_T	m_T is a parameter that amplifies the hypoplastic stiffness to account for the high tangent stiffness during a 90-degree change of loading direction. Assuming a linear change of stiffness with change of direction (for instance according to Atkinson *et al*’s [22] data $$m_T = \frac{m_R + 1}{2} = 3$$ Niemunis and Herle (1997) recommended $m_T = 2$	
$\chi\ (> 1)$	This parameter describes the nonlinearity of the decay of stiffness with the intergranular strain stiffness with the magnitude of the intergranular strain. For a given limit small strain, the choice of χ must be combined with the choice of β_r as given in the following row. Some suggested values in the literature are summarized in a table in [4].	
β_r	This parameter describes the nonlinearity of the evolution of the intergranular strain rate with its magnitude.	

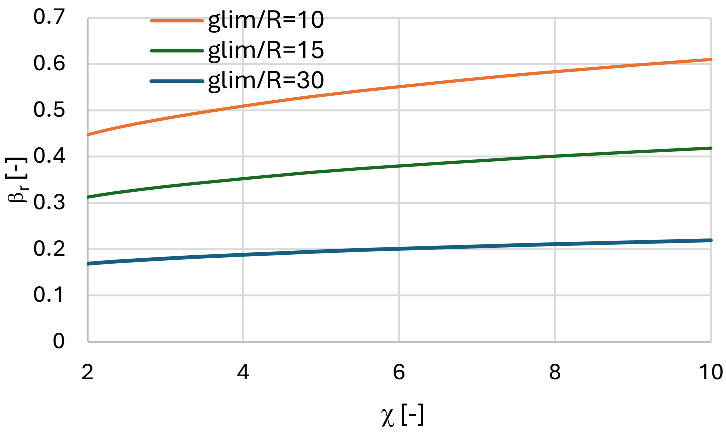
Supposing γ_{lim} to be the swept out of memory strain (where the intergranular strain is reduced from an active player to a watcher)

The following approximate relation was derived in Tsegaye *et al*. 2010 [36], Tsegaye and Benz (2014) [37]

$$\gamma_{lim} \approx 3.44R\chi^{0.233}\beta_r^{0.033\ln\chi-1.15}$$

A set of curves relating parameter χ and β_r for different values of γ_{lim}/R are given in the following chart.

Since the degradation curve is effective confining pressure dependent β_r may also be effective confining pressure. Such formulation can be found in [33].



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