Understanding theory of consolidation and some practical applications

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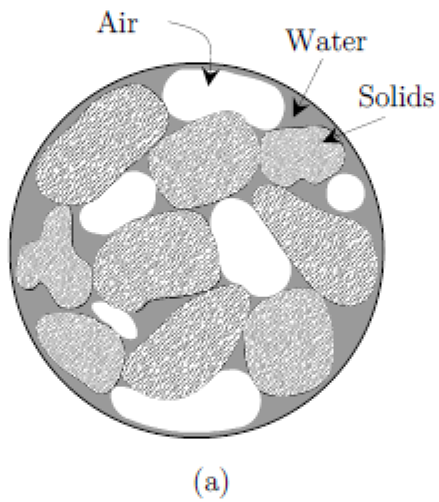
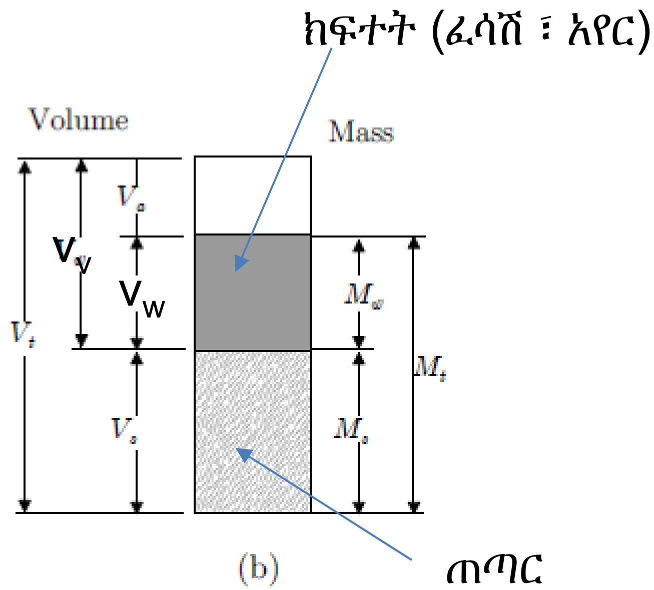
Content

- Elementary mechanical behaviour of soils
- Generation and dissipation of pore pressures
- Engineering problems related to generation and dissipation of pore pressures
- Theory of consolidation
- Back to engineering… how to use the knowledge of consolidation to solve engineering problems related to generation and dissipation of pore pressures
- Asaoka’s observational method

Elementary mechanical behaviour of soils (የዐፈር መሠረታዊ የሆኑ ሜካኒካዊ ባሕርያት)

- Phase relationships (የይዘት ዝምድናዎች)





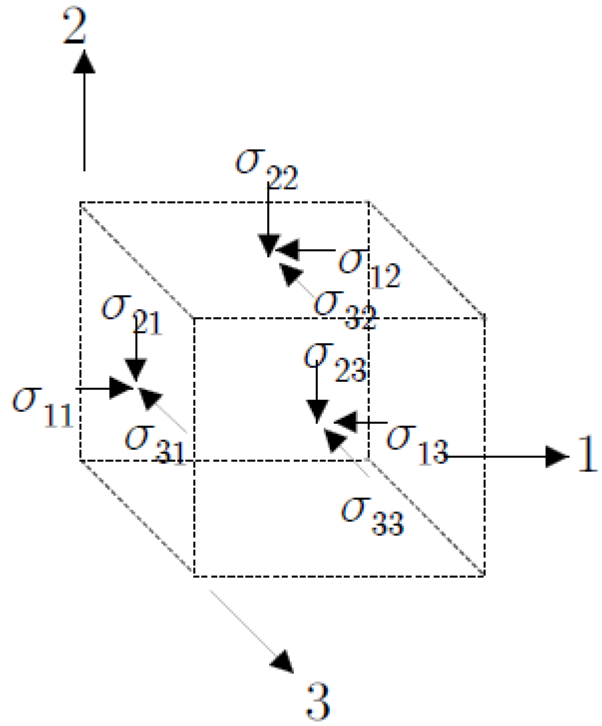
$$Void\ ratio\ (\text{ውድር ክፍተት})e = \frac{Volume\ of\ voids(\text{የክፍተት ይዘት})}{Volume\ of\ solids(\text{የጠጣር ይዘት})}$$

Elementary mechanical…

- Stress defn:
  - ❑ $Stress(\sigma) = \frac{Force(F)}{Area(A)}$

- Terzaghi's effective stress principle:
  - ❑ $Total\ stress\ (\sigma) =$
    $effective\ stress\ (\sigma') +$
    $pore\ water\ pressure\ (p)$

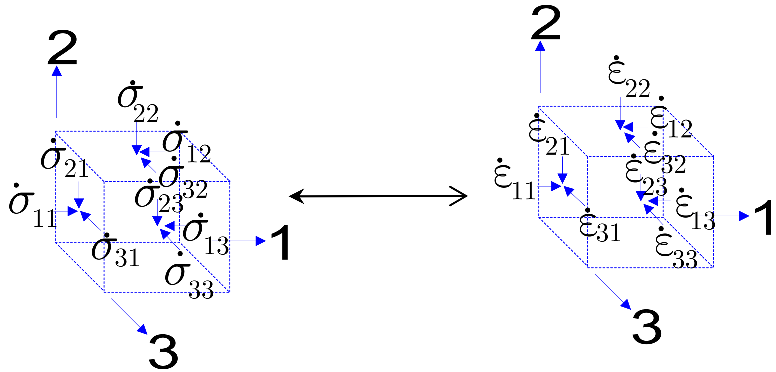
- Further decomposition of effective stress
  - ❑ $Total\ stress\ (\sigma) =$
    $effective\ confinign\ pressure\ (p') +$
    $deviatoric\ stress\ (q) +$
    $pore\ water\ pressure\ (p)$



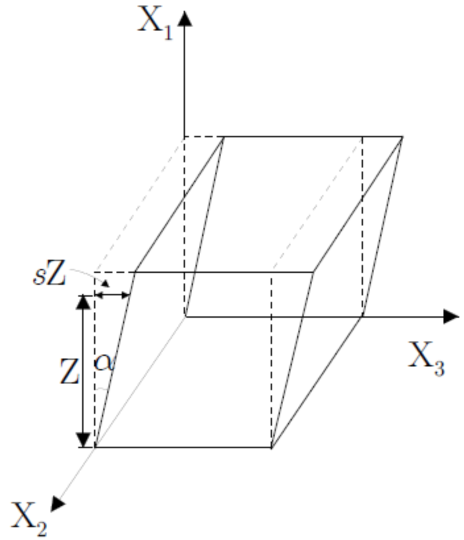
አማርኛ፡ ጫና = መጠነ ግደት በመጠነ ስፋት
ጠቅላላ ጫና = ቅንጣት ቅንጣት ጫና + የውኃ ግፊት
ቅንጣት ቅንጣት ጫና =አማቂ ጫና + አንሸራታች ጫና

Elementary mechanical…

- Strain defn:
  - ❑ $Strain(\varepsilon) = \frac{Change\ in\ length(\Delta l)}{Length(L)}$
- Further decomposition of strain
  - ❑ $Strain\ (\varepsilon) = volumetric\ strain\ (\varepsilon_v) + deviatoric\ strain\ (\varepsilon_q)$
- Stresses cause strains
  - ❑ Effective stress increment ($\Delta\boldsymbol{\sigma}'$)= stiffness($\boldsymbol{D}$) x strain increment ($\Delta\boldsymbol{\varepsilon}$)



ሁክ: «ceiiinosssttuv» - Ut tensio, sic vis - መጠነ ጫና እንደ መጠነ ውጥረት ነው።



የንሽርትት ውጥረት (ውልጫሜ)

አማርኛ: ውጥረት = ለውጥ ርዝመት በርዝመት

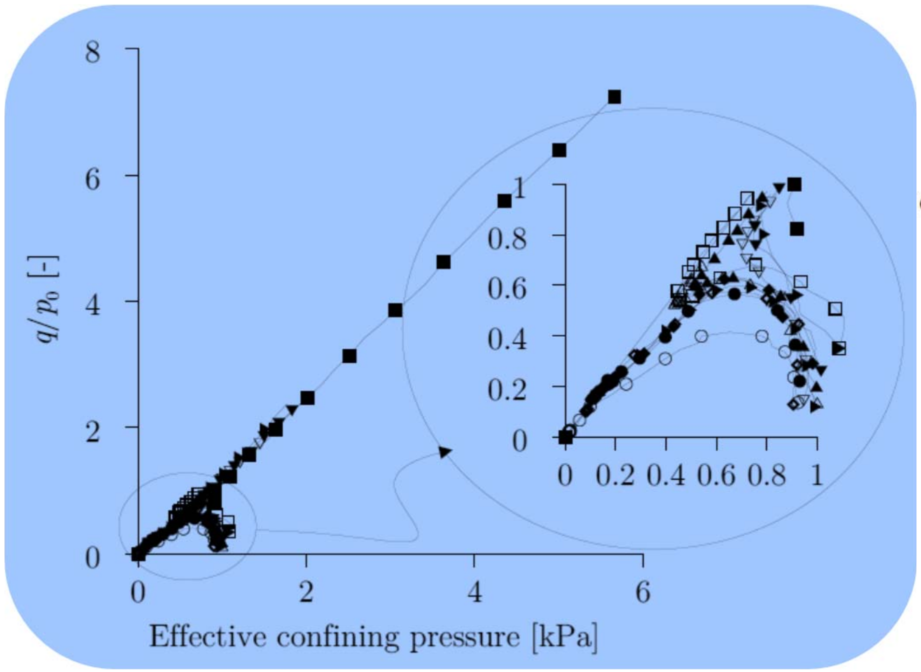
ውጥረት = የመጠን ውጥረት + የእንሽርትት ውጥረት (ውልጫሜ)

Elementary mechanical…

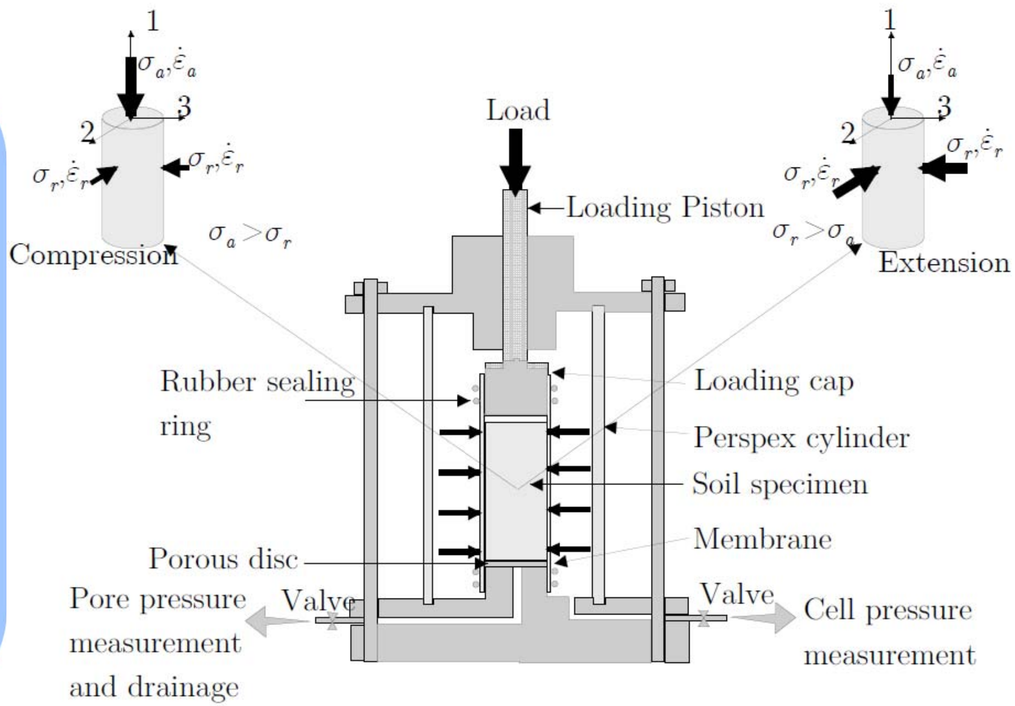
- $\tau_s = c + \sigma \tan\varphi$ (Coulomb 1776)
- $\tau_s = c + \sigma' \tan\varphi_c + \sigma' \tan\psi$ (Taylor 1948)
  - The shear strength of soil comes from interparticle cohesion, interparticle friction and interlocking
  - የዐፈር ሸርተታ የመቋቋም አቅም (ሸከመ ሸርተታ) የሚገኘው ከቅንጣቶች **መጣበቅ** ፣ በቅንጣቶች መካከል ከሚኖር **ፍትጊያ** እና ከቅንጣቶች **መቆላለፍ** ነው።
  - Interparticle contact increases with increasing effective confining pressure and decreases otherwise
  - አማቂ ጫና ሲጨመር የቅንጣቶች ንክኪ ይጨምራል ፣ ፣ ንክኪያቸው ሲጨምር ፍትጊያቸው እና መቆላለፋቸው ይጨምራል ፣ ሲቀንስ ደግሞ ይቀንሳል
  - Pore pressure increase leads to decreasing effective confining pressure and thus reduced shear strength
  - ስለዚህም የክፍተት ውኃ ግፊት ሲጨመር የቅንጣት ቅንጣት አማቂ ጫና መቀነስን ያስከትላል ፣ ይኸም ሸርተታ የመቋቋም አቅም መቀነስን ያመጣል።

Generation of pore pressure (የክፍተት ውኃ ግፊት ማሻቀብ)

- Undrained monotonic loading of loose/contractive soils (ዝግ ወጥ አቅጣጫ ጫና)

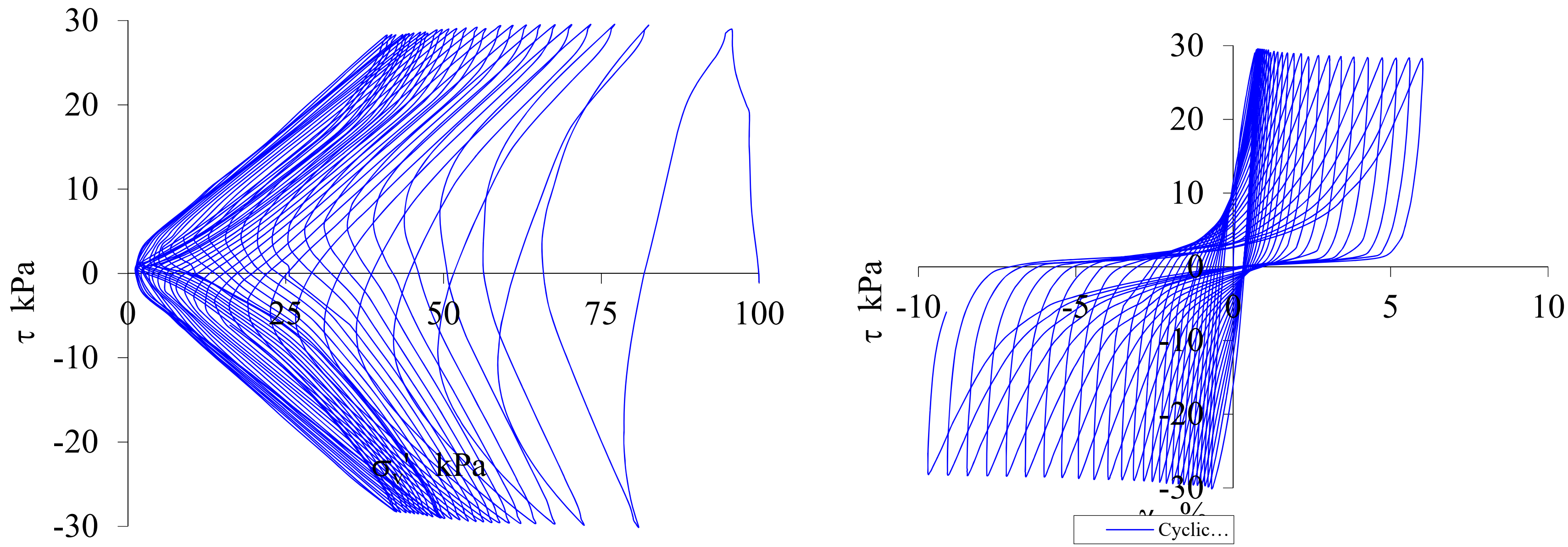


Undrained Triaxial compression test
(ማስተንፈሻ አልባ የዐምዳዊ ዕምቀት ፍተሻ)



Generation of pore pressure (የክፍተት ውኃ ግፊት ማሻቀብ)

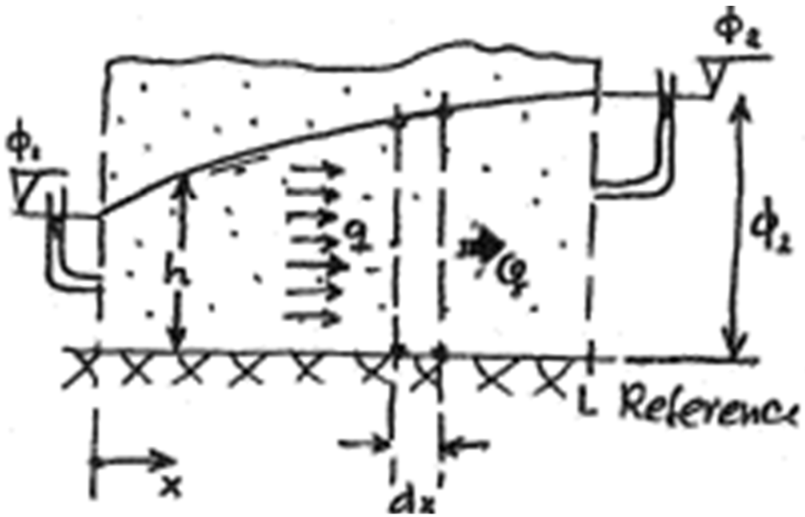
- Undrained cyclic loading (ዝግ ዐውዳዊ ፣ ተመላላሽ ጫና)



Sriskandakumar (2004)

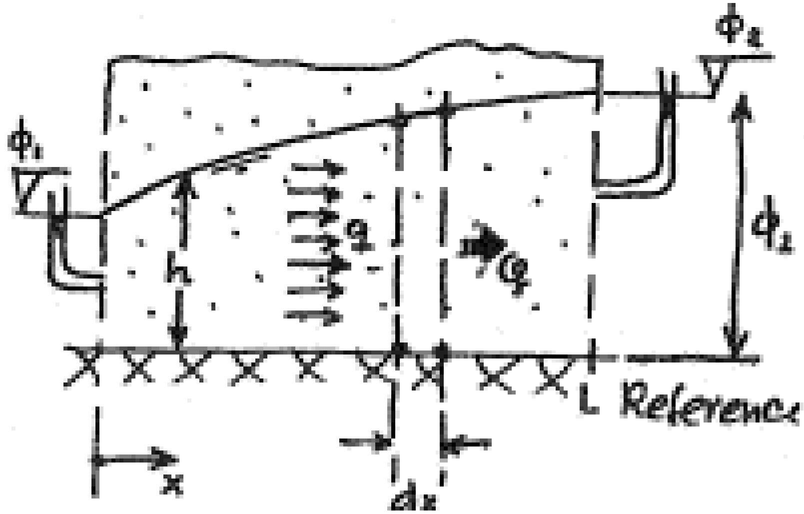
Dissipation of pore pressure (የክፍተት ውኃ ግፊት ማቆልቆል)

- **Darcy's flow**: Discharge is linearly proportional with pressure gradient. Continued discharge intern leads to reduction in pressure – which we call dissipation of pore pressure.
- **የዳርሲ ፍሰት፡** በሁለት ሥፍራዊ ነጥቦች መካከል የሚደረግ መጠነ ፍሳሽ በሁለቱ ነጥቦች መካከል ካለው የግፊት ተዳፋት ጋር በቀጥታ ወደረኛ ነው። በግልባጩም ፣ ፈሳሽ አስተላላፊ በሆነ መሆኛ ፣ በሁለት ሥፍራዊ ነጥቦች መካከል ያለ የግፊት ልዩነት ፍሰት ያስከትላል። በሒሳብ ሐረግ ሲቀመጥ
  - $q = \frac{k}{\gamma_w} \nabla p, \nabla = \frac{\partial}{\partial x}$



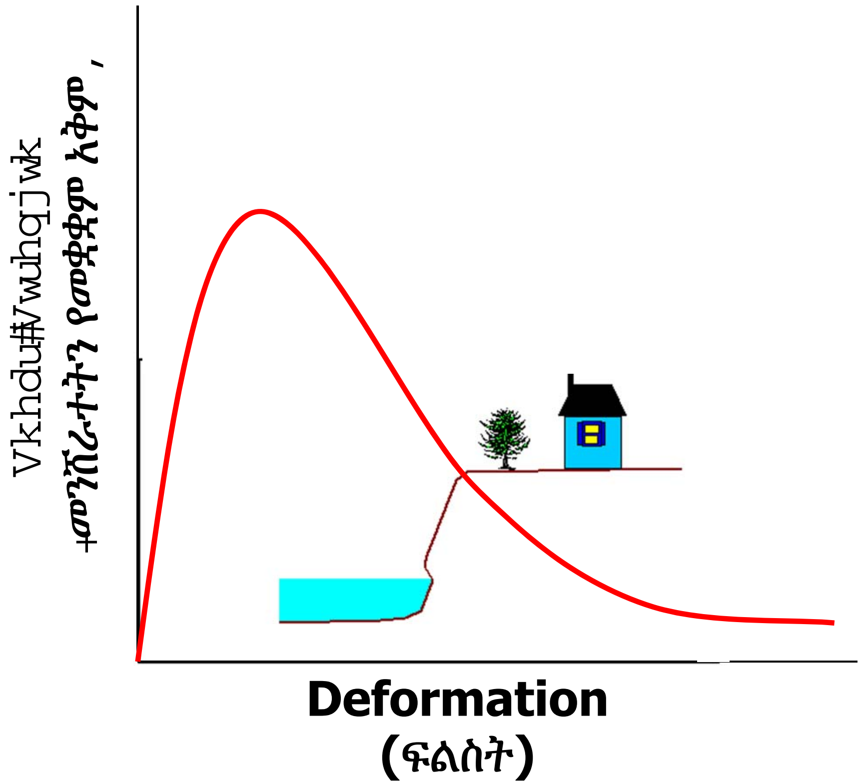
Dissipation of pore pressure (የክፍተት ውኃ ግፊት ማቆልቆል)

- **Consolidation (ስብሳቦሽ)**- generated pore pressure can dissipate if the pore fluid can escape from high pressure areas into low pressure areas leaving the grains to move closer to each other there by reducing the pore space between them.
- **ስብሳቦሽ፡**- ግፊት ያዘለው የክፍተት ውኃ ከከፍተኛ ግፊት ወደ ዝቅተኛ ግፊት በመንቀሳቀሱ ምክንያት በቅንጣቶች መካከል ያለው የክፍተት መቀነስ እና የቅንጣቶች ጥግጊ መጨመር የሚመጣ ነው።



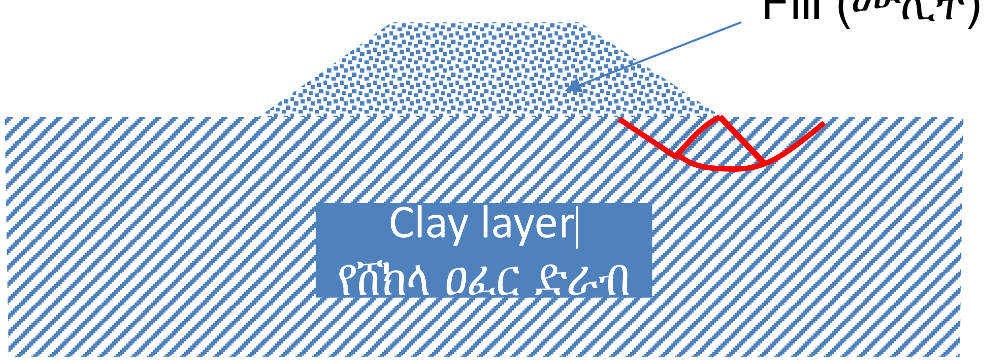
Engineering problems due to generation of excess pore pressure

- Slope stability, liquefaction (የተዳፋት ማሽቅሽቅ ፣ መንሸራተት ፣ ፍሰት)



Engineering problems due to generation of excess pore pressure

- Embankment failure (የግድብ ቁልል መንሸራተት)

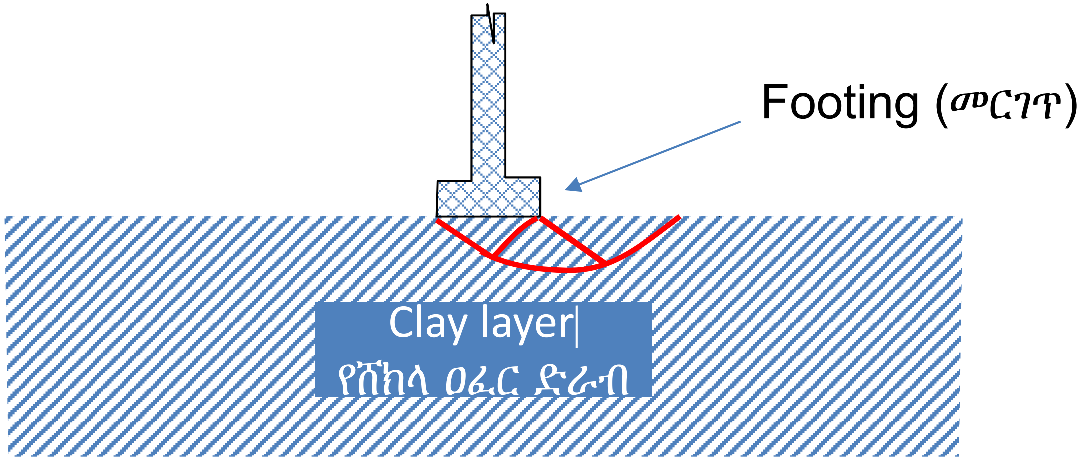


$$\Delta\sigma = \Delta\sigma'\downarrow + \Delta p\uparrow$$

Pore pressure increase leads to decreasing effective confining pressure and thus reduced shear strength
የክፍተት ውኃ ግፊት ሲጨመር የቅንጣት ቅንጣት አማቂ ጫና መቀነስን ያስከትላል ፣ ይኸም ሸርተታ የመቋቋም አቅም መቀነስን ያመጣል።

Engineering problems due to generation of pore pressure

- Bearing capacity failure (የመሠረት ተሸካሚ ዐፈር መንሸራተት)

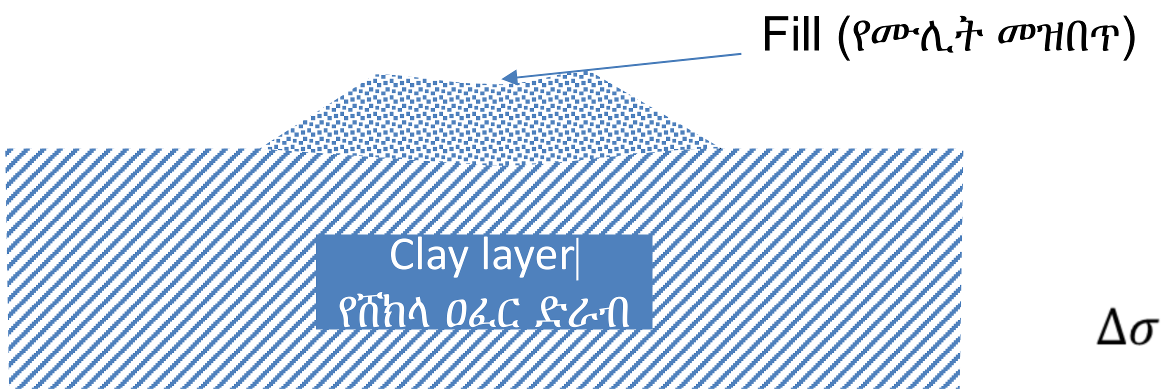


$$\Delta\sigma = \Delta\sigma'\downarrow + \Delta p \uparrow$$

Pore pressure increase leads to decreasing effective confining pressure and thus reduced shear strength
የክፍተት ውኃ ግፊት ሲጨመር የቅንጣት ቅንጣት አማቂ ጫና መቀነስን ያስከትላል ፣ ይኸም ሸርተታ የመቋቋም አቅም መቀነስን ያመጣል።

Engineering problems due to dissipation of pore pressure

- Embankment (ቁልል ግድብ)



$$\Delta\sigma = \Delta\sigma'\downarrow + \Delta p\uparrow$$

Pore pressure increase leads to decreasing effective confining pressure and thus reduced shear strength
የክፍተት ውኃ ግፊት ሲጨመር የቅንጣት ቅንጣት አማቂ ጫና መቀነስን ያስከትላል ፣ ይኸም ሸርተታ የመቋቋም አቅም መቀነስን ያመጣል።

Engineering problems due to dissipation of pore pressure

- Foundation (መሠረት)- Uniform settlement (ወጥ ዝብጠት)

$$\Delta\sigma = \Delta\sigma' \uparrow + \Delta p \downarrow$$

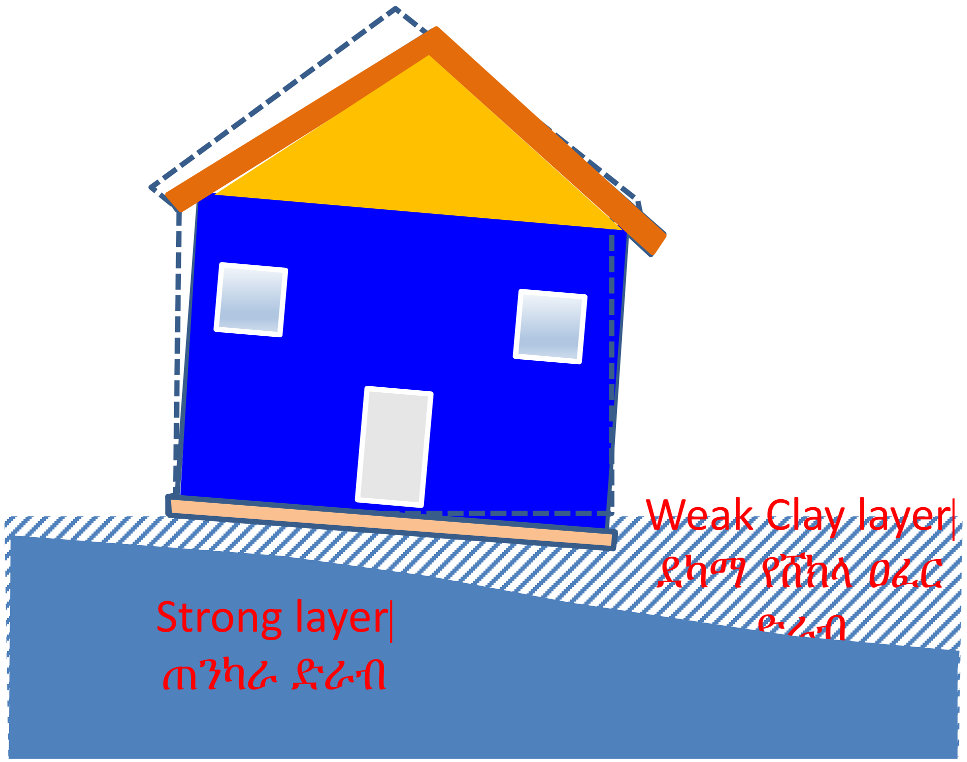
Engineering problems due to dissipation of pore pressure

- Foundation (መሠረት)- Differential settlement (የዝብጠት ልዩነት)

$$\Delta\sigma = \Delta\sigma'\uparrow + \Delta p \downarrow$$

Engineering problems due to dissipation of pore pressure

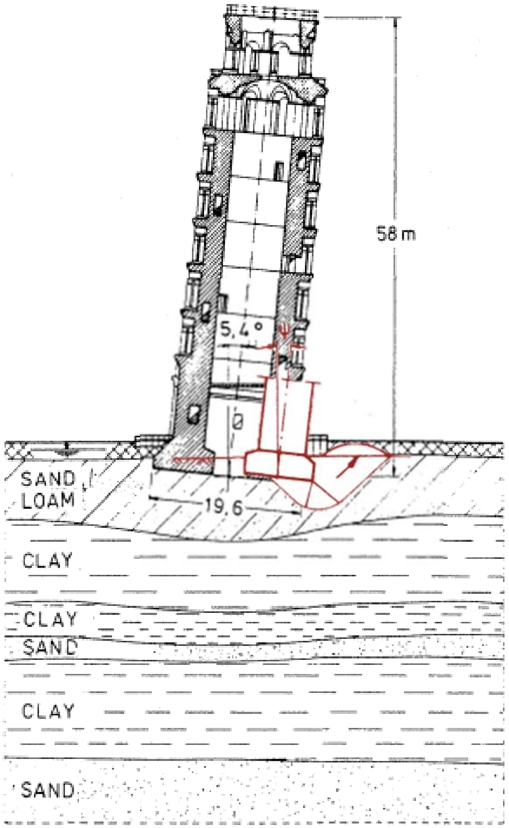
- Foundation (መሠረት)-Tipping settlement (ማዘንበል ፣ ማገዘመም)



$$\Delta\sigma = \Delta\sigma'\uparrow + \Delta p\downarrow$$

Engineering problems due to dissipation of pore pressure

- The leaning tower of Pisa (ያጋደለው የፒሳ ፎቅ)



Grabe and Gudehus (1992)

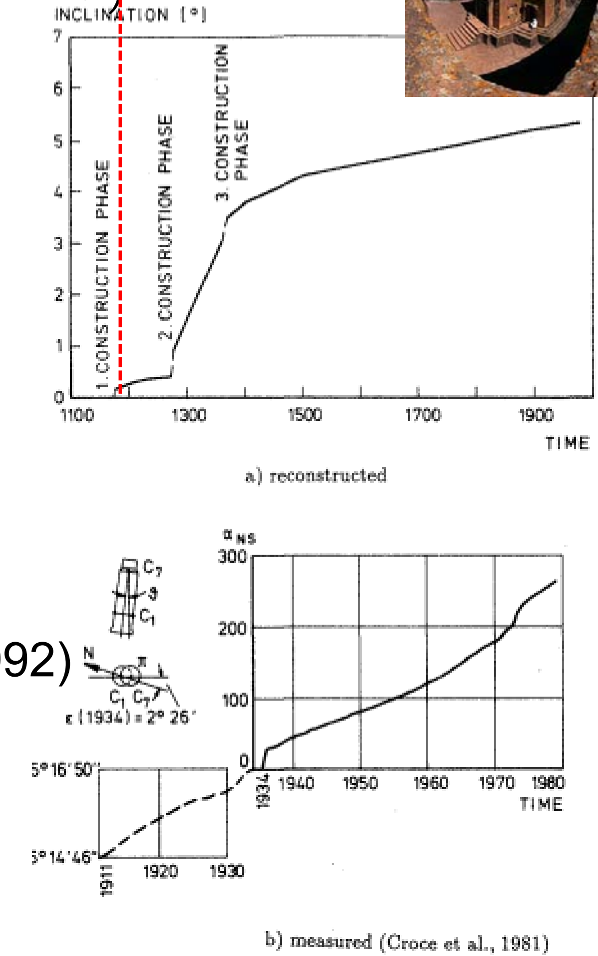
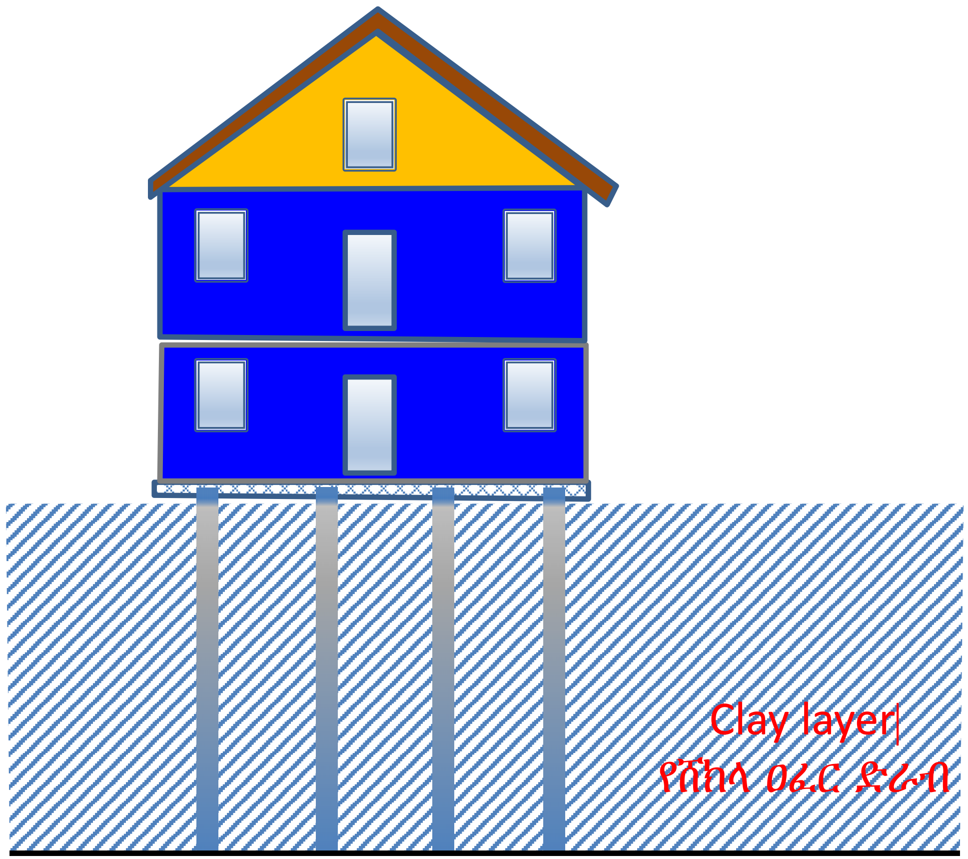


Fig. 2. Development of the inclination with time

Tipping settlement?/ bearing capacity failure?

Engineering problems due to dissipation of pore pressure

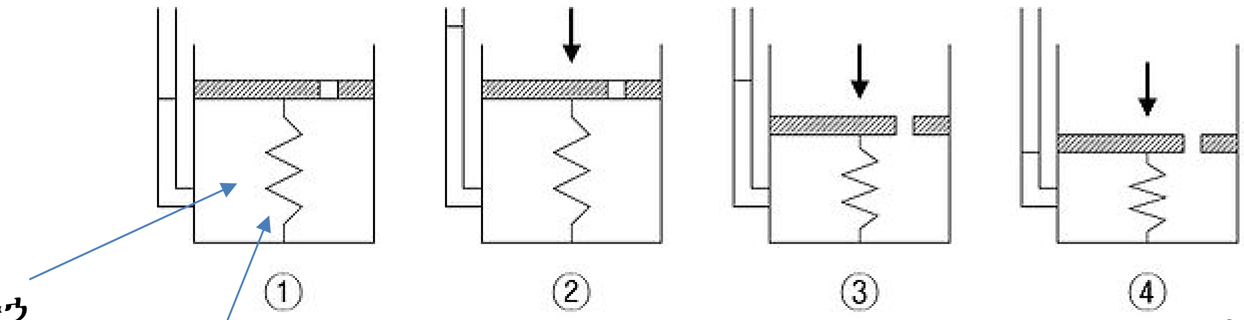
- Foundation (መሠረት)- Overhanging loads (ተንጠልጣይ ጭነቶች)



$$\Delta\sigma = \Delta\sigma'\uparrow + \Delta p \downarrow$$

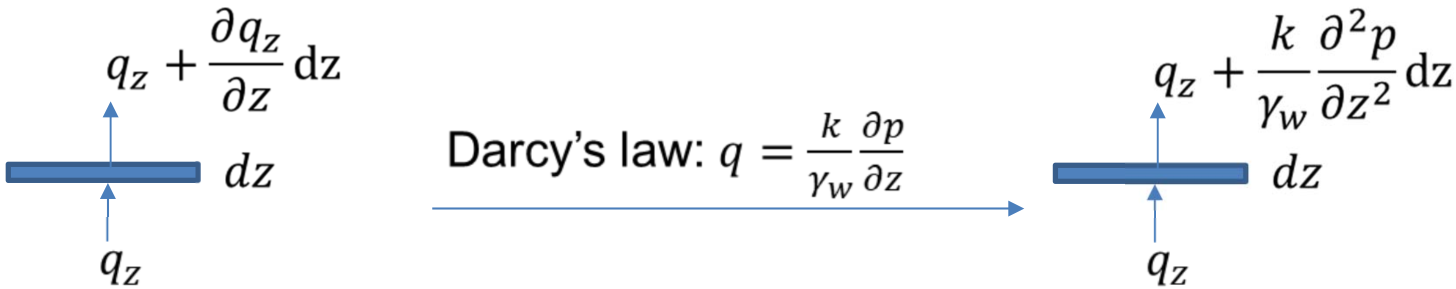
Theory of consolidation
የስብስቦሽ ንድፈ ሐሳብ

- Spring-water system



Continuity & Consolidation

Let us disregard compressibility of pore water and consider volume change due to flow:



The net volume change ($\mathrm{d}Q$):

- $\mathrm{d}Q = -\frac{k}{\gamma_w} \frac{\partial^2 p}{\partial z^2} \mathrm{dzA} = -\frac{k}{\gamma_w} \frac{\partial^2 p}{\partial z^2} \Omega$

- $\frac{\mathrm{d}Q}{\Omega} = -\frac{k}{\gamma_w} \frac{\partial^2 p}{\partial z^2}$

- $\frac{1}{\Omega} \frac{\mathrm{d}\Omega}{dt} = \frac{d\varepsilon}{dt} = -\frac{k}{\gamma_w} \frac{\partial^2 p}{\partial z^2}$

Continuity & Consolidation

The net volume change (Darcy flow):

- ❑ $dQ = -\frac{k}{\gamma_w}\frac{\partial^2 p}{\partial z^2}\,\mathrm{dzA} = -\frac{k}{\gamma_w}\frac{\partial^2 p}{\partial z^2}\Omega$

- ❑ $\frac{dQ}{\Omega} = -\frac{k}{\gamma_w}\frac{\partial^2 p}{\partial z^2}$

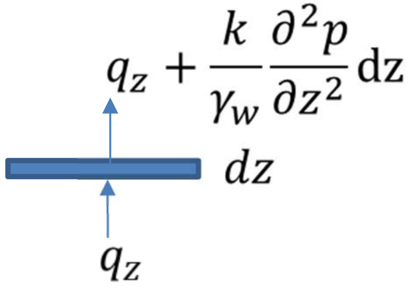
- ❑ $\frac{1}{\Omega}\frac{d\Omega}{dt} = \frac{d\varepsilon}{dt} = -\frac{k}{\gamma_w}\frac{\partial^2 p}{\partial z^2}$

Hook (*ceiiinosssttuv* - as the extension so is the force)

- ❑ $\frac{\partial\varepsilon}{\partial t} = \frac{1}{m_v}\frac{\partial\sigma\prime}{\partial t}$

Terzaghi

- ❑ $\frac{\partial\varepsilon}{\partial t} = \frac{1}{m_v}\frac{\partial(\sigma-p)}{\partial t} = -\frac{1}{m_v}\frac{\partial p}{\partial t}$



$$\frac{\partial p}{\partial t} = c_v\frac{\partial^2 p}{\partial z^2}$$

$$c_v = \frac{km_v}{\gamma_w}$$

ሁለተኛ ርከን የልውጠት እኩልዮሽ

Dimensionless analysis

$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial z^2}$$

- Normalizing various variables (Barends,2008)

$$\frac{\partial^2 (p/p_0)}{\partial (z/H)^2} \Big/ \frac{\partial (p/p_0)}{\partial (t/T)} = \frac{H^2}{c_v T} = \frac{1}{N}$$

Flow term

Storage term

Question: What does it mean when

a. N is significantly smaller than one?

b. N is significantly higher than one?

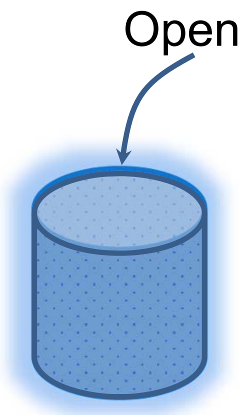
Value of N	Small(<<1)	=1	>>1
Implication	The flow factor is large compared to the storage factor	The flow factor is equal to the storage factor	The flow factor is small compared to the storage term
Process stage	Start of consolidation	Consolidation in full process	End of consolidation

Approximate solution to the 1D consolidation equation

$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial z^2}$$

Storage

1. $= \int_\Omega \frac{\partial p}{\partial t} d\Omega$
2. $= \Omega \frac{\partial}{\partial t}\left(\frac{1}{\Omega}\int_\Omega p d\Omega\right)$
3. $= \Omega \frac{\partial \bar{p}}{\partial t}$



Flow

1. $= c_v \int_V \frac{\partial^2 p}{\partial z^2} d\Omega$
2. $= c_v \frac{\gamma_w}{k} \int_V \frac{\partial q}{\partial z} d\Omega$
3. $= c_v \frac{\gamma_w}{k} \int_S q dS$
4. $= -c_v \int_S \frac{\partial p}{\partial z_b} dS$

❑ $\Omega \frac{\partial \bar{p}}{\partial t} = -c_v \int_S \frac{\partial p}{\partial z_b} dS$

Approximate solution to the 1D consolidation equation

$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial z^2}$$

$$\Omega \frac{\partial \bar{p}}{\partial t} = -c_v \int_S \frac{\partial p}{\partial z}_b \, dS$$

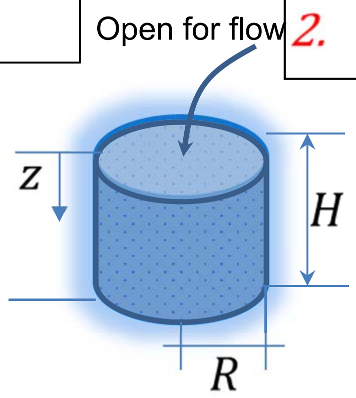
Storage

Flow

Let $p = p_0 f(z) f(t)$

1. $\bar{p} = \frac{1}{\pi R^2 H} \int_V p_0 f(z) f(t) 2\pi r dr dz$
2. $\Omega \frac{\partial \bar{p}}{\partial t} = \pi R^2 p_0 \frac{\partial f(t)}{\partial t} \int_0^H f(z) dz$

1. $= -2\pi p_0 c_v f(t) \left(\frac{\partial f(z)}{\partial z}\right)_{z=0} \int_S r dr$
2. $-\pi R^2 p_0 c_v f(t) \left(\frac{\partial f(z)}{\partial z}\right)_{z=0}$



- ❑ $\frac{\partial f(t)}{\partial t} \int_0^H f(z) dz = -c_v f(t) \left(\frac{\partial f(z)}{\partial z}\right)_{z=0}$
- ❑ $\frac{df(t)}{f(t)} = -c_v \left(\frac{\partial f(z)}{\partial z}\right)_{z=0} dt / \int_0^H f(z) dz$

Approximate solution to the 1D consolidation equation

$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial z^2}$$

$$\Omega \frac{\partial \bar{p}}{\partial t} = -c_v \int_S \frac{\partial p}{\partial z} dS$$

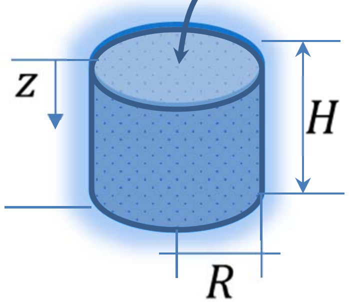
Storage

Flow

Let $p = p_0 f(z) f(t)$

1. $\bar{p} = \frac{1}{\pi R^2 H} \int_V p_0 f(z) f(t) 2\pi r dr dz$
2. $\Omega \frac{\partial \bar{p}}{\partial t} = \pi R^2 p_0 \frac{\partial f(t)}{\partial t} \int_0^H f(z) dz$

1. $= -2\pi p_0 c_v f(t) \left(\frac{\partial f(z)}{\partial z}\right)_{z=0} \int_S r dr$
2. $-\pi R^2 p_0 c_v f(t) \left(\frac{\partial f(z)}{\partial z}\right)_{z=0}$



- $\int_1^{f(t)} \frac{df(t)}{f(t)} = -\int_0^t \frac{c_v \left(\frac{\partial f(z)}{\partial z}\right)_{z=0}}{\int_0^H f(z) dz} dt$

Approximate solution to the 1D consolidation equation

$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial z^2}$$

$$\Omega \frac{\partial \bar{p}}{\partial t} = -c_v \int_S \frac{\partial p}{\partial z} dS$$

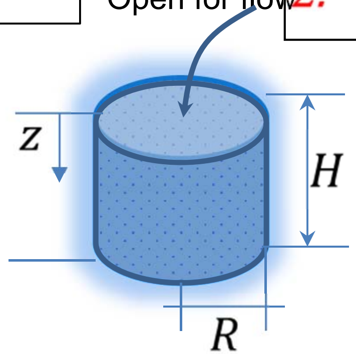
Let $p = p_0 f(z) f(t)$

Storage

1. $\bar{p} = \frac{1}{\pi R^2 H} \int_V p_0 f(z) f(t) 2\pi r dr dz$
2. $\Omega \frac{\partial \bar{p}}{\partial t} = \pi R^2 p_0 \frac{\partial f(t)}{\partial t} \int_0^H f(z) dz$

Flow

1. $= -2\pi p_0 c_v f(t) \left(\frac{\partial f(z)}{\partial z}\right)_{z=0} \int_S r dr$
2. $-\pi R^2 p_0 c_v f(t) \left(\frac{\partial f(z)}{\partial z}\right)_{z=0}$



❑ $\ln f(t) = -\frac{c_v \left(\frac{\partial f(z)}{\partial z}\right)_{z=0}}{\int_0^H f(z) dz} t$

Approximate solution to the 1D consolidation equation

$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial z^2}$$

$$\Omega \frac{\partial \bar{p}}{\partial t} = -c_v \int_S \frac{\partial p}{\partial z} dS$$

Storage

Flow

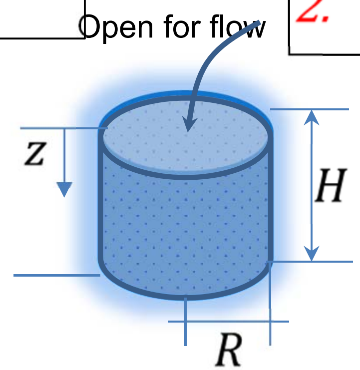
Let $p = p_0 f(z) f(t)$

Storage:

1. $\bar{p} = \frac{1}{\pi R^2 H} \int_V p_0 f(z) f(t) 2\pi r dr dz$
2. $\Omega \frac{\partial \bar{p}}{\partial t} = \pi R^2 p_0 \frac{\partial f(t)}{\partial t} \int_0^H f(z) dz$

Flow:

1. $= -2\pi p_0 c_v f(t) \left(\frac{\partial f(z)}{\partial z}\right)_{z=0} \int_S r dr$
2. $-\pi R^2 p_0 c_v f(t) \left(\frac{\partial f(z)}{\partial z}\right)_{z=0}$



- ❑ $f(t) = exp\left\{-\frac{c_v \left(\frac{\partial f(z)}{\partial z}\right)_{z=0}}{\int_0^H f(z) dz} t\right\}$

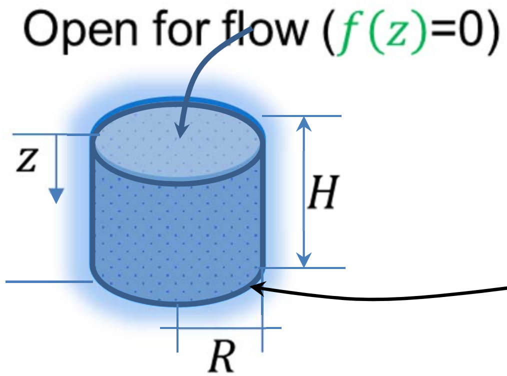
Approximate solution to the 1D consolidation equation

$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial z^2}$$

$$\Omega \frac{\partial \bar{p}}{\partial t} = -c_v \int_S \frac{\partial p}{\partial z} dS$$

Storage

Flow



$$\frac{\partial f(z)}{\partial z} = 0$$

$$p(z,t) \overset{?}{=} p_0 f(z) exp\left\{-\frac{c_v \left(\frac{\partial f(z)}{\partial z}\right)_{z=0}}{\int_0^H f(z)dz} t\right\}$$

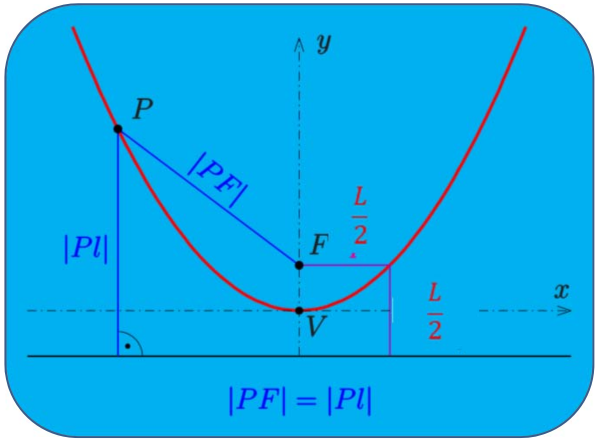
Approximate solution to the 1D consolidation equation

$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial z^2}$$

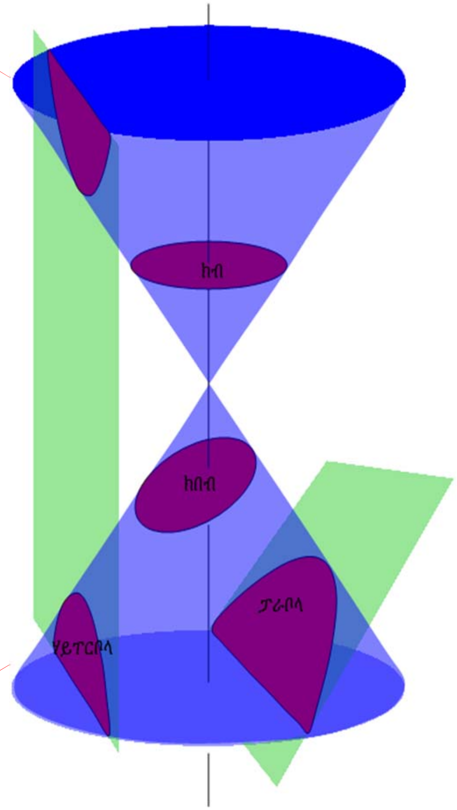
$$\Omega \frac{\partial \bar{p}}{\partial t} = -c_v \int_S \frac{\partial p}{\partial z} dS$$

Storage

Flow



$$Ly = x^2$$



?

$$p(z,t) = p_0 f(z) exp\left\{ -\frac{c_v \left(\frac{\partial f(z)}{\partial z}\right)_{z=0}}{\int_0^H f(z)dz} t \right\}$$

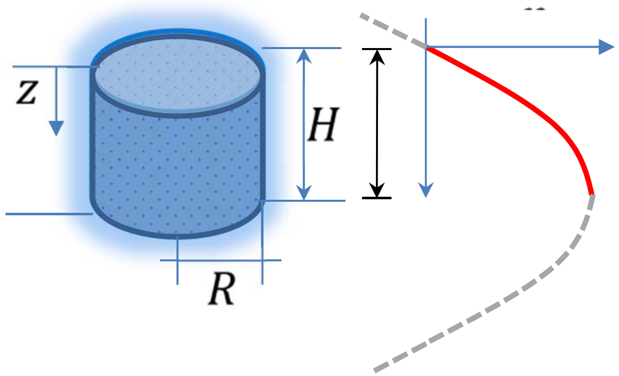
Approximate solution to the 1D consolidation equation

$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial z^2}$$

$$\Omega \frac{\partial \bar{p}}{\partial t} = -c_v \int_S \frac{\partial p}{\partial z} dS$$

Storage

- ❑ $Lp = H^2 - (H-z)^2$
- ❑ $p_{z=H} = \frac{H^2}{L}$
- ❑ $\frac{p}{p_{z=H}} = 1 - \left(1 - \frac{z}{H}\right)^2$



Flow

- ❑ $f(z) = 1 - \left(1 - \frac{z}{H}\right)^2$
- ❑ $\frac{\partial f(z)}{\partial z} = \frac{2\left(1 - \frac{z}{H}\right)}{H}$

$$p(z,t) = p_0 f(z) exp\left\{ -\frac{c_v \left(\frac{\partial f(z)}{\partial z}\right)_{z=0}}{\int_0^H f(z)dz} t \right\}$$

Approximate solution to the 1D consolidation equation

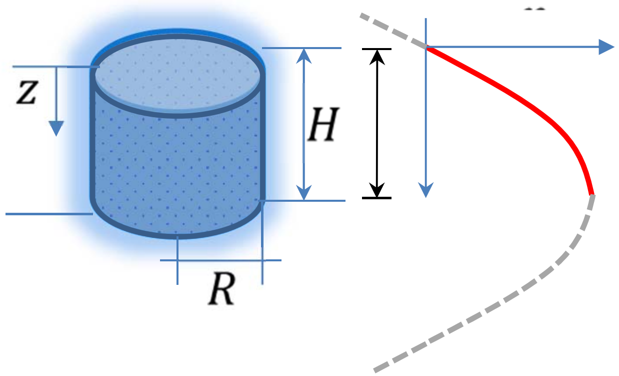
$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial z^2}$$

$$\Omega \frac{\partial \bar{p}}{\partial t} = -c_v \int_S \frac{\partial p}{\partial z} dS$$

Storage

Flow

- ❑ $f(z) = 1 - \xi^2, \ \xi = 1 - \frac{z}{H}$
- ❑ $\int_0^H f(z)dz = \frac{2}{3}H$
- ❑ $\frac{\partial f(z)}{\partial z} = \frac{2\xi}{H}$



$$p(z,t) = p_0 f(z) exp\left\{ -\frac{c_v \left(\frac{\partial f(z)}{\partial z}\right)_{z=0}}{\int_0^H f(z)dz} t \right\}$$

Approximate solution to the 1D consolidation equation

$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial z^2}$$

$$\Omega \frac{\partial \bar{p}}{\partial t} = -c_v \int_S \frac{\partial p}{\partial z} dS$$

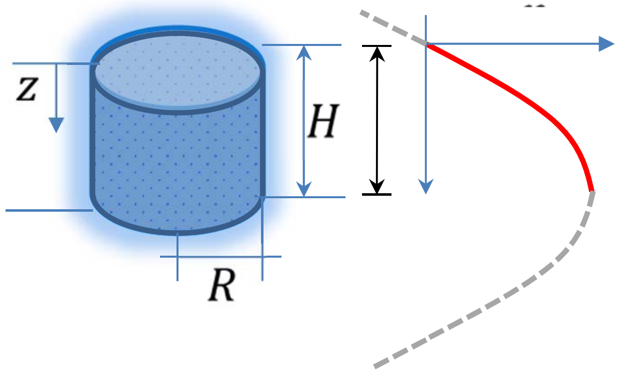
Storage

Flow

- $f(z) = 1 - \xi^2$

- $\int_0^H f(z)dz = \frac{2}{3}H$

- $\frac{\partial f(z)}{\partial z} = \frac{2\xi}{H}$



$$p(z,t) = p_0\{1-\xi^2\}exp\left\{-3\frac{c_v}{H^2}t\right\}$$

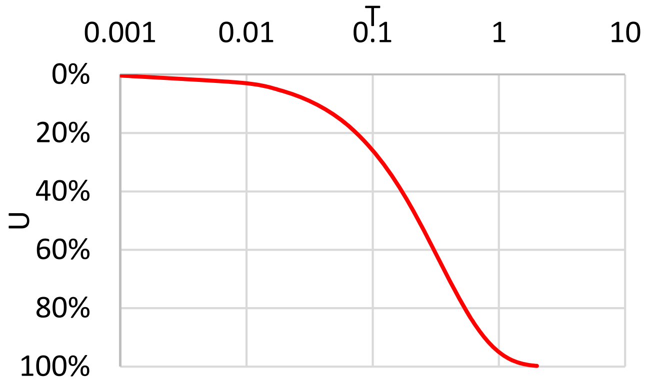
Approximate solution to the 1D consolidation equation

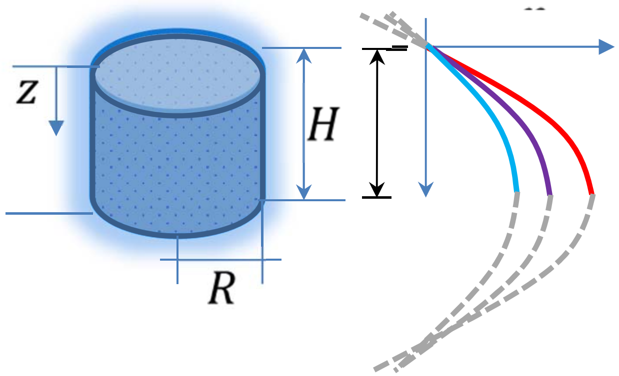
$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial z^2}$$

$$\Omega \frac{\partial \bar{p}}{\partial t} = -c_v \int_S \frac{\partial p}{\partial z} dS$$

Storage

Flow





$$T = \frac{c_v}{H^2} t$$

- $U = 1 - \frac{p(z,t)}{p(z,0)} = 1 - \exp(-3T)$

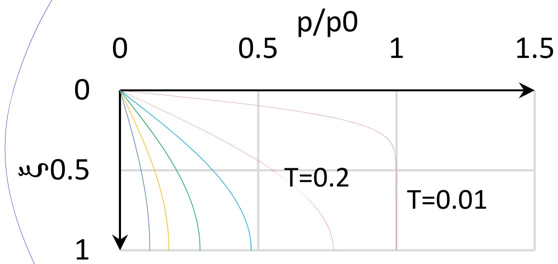
Terzaghi's exact solution to the 1D consolidation equation

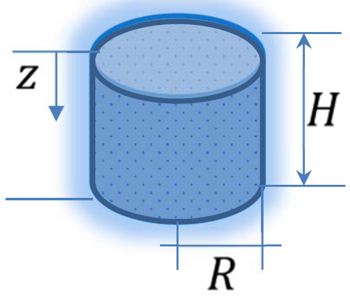
Storage

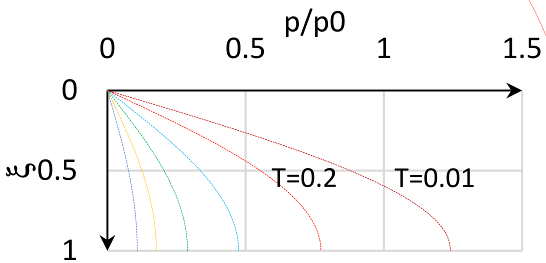
$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial z^2}$$

Flow

$$T = \frac{c_v}{H^2} t \qquad \xi = \frac{H - z}{H}$$







$$p(\xi, T)/p_0 = \frac{4}{\pi} \cos\left[\frac{\pi}{2}\xi\right] \exp\left[-\left(\frac{\pi}{2}\right)^2 T\right]$$

$$p(\xi, T)/p_0 = \frac{4}{\pi} \sum \frac{(-1)^{j-1}}{2j-1} \cos\left[\frac{\pi}{2}(2j-1)\xi\right] \exp\left[-\left(\frac{\pi}{2}(2j-1)\right)^2 T\right]$$

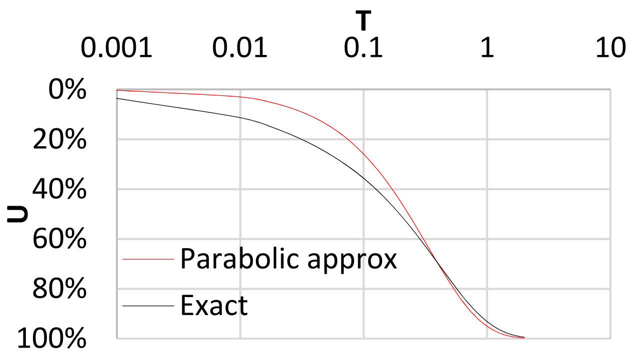
Exact solution to the 1D consolidation equation

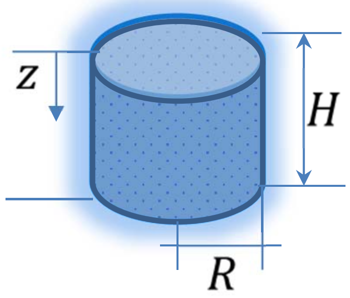
Storage

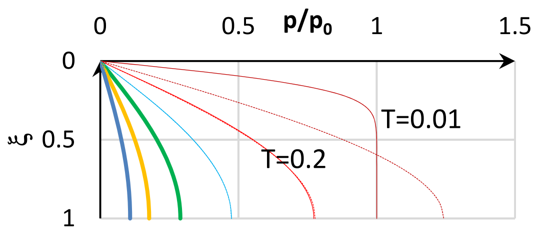
$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial z^2}$$

Flow

$$T = \frac{c_v}{H^2} t \qquad \xi = \frac{H - z}{H}$$







- $U = \frac{s}{s_\infty} = \int_0^H \varepsilon dz / \int_0^H \varepsilon_\infty dz$

$$= 1 - \frac{\pi^2}{8} \sum (2j-1)^2 \exp\left\{-\left(\frac{\pi}{2}(2j-1)\right)^2 T\right\}$$

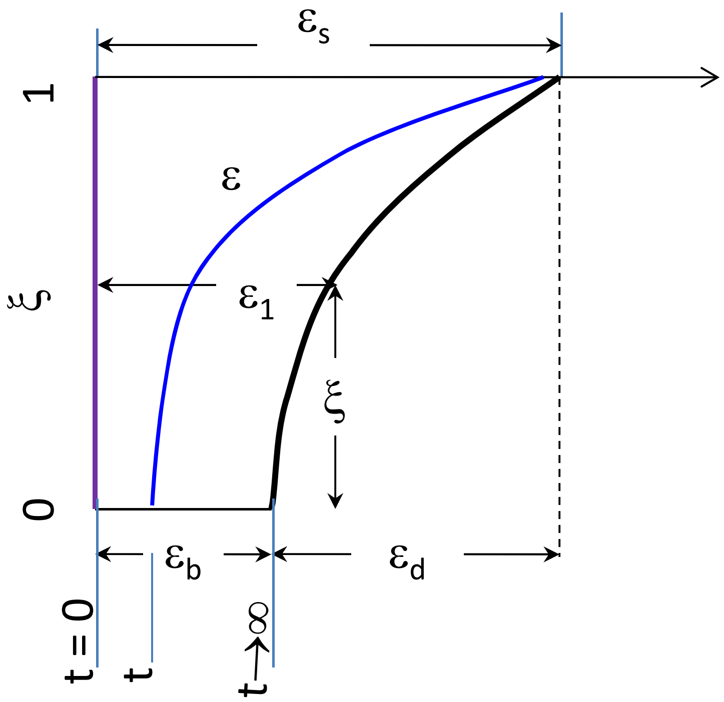
Some limitations of the classical 1DC Theory

❑ $p(\xi, T)/p_0 = \frac{4}{\pi} \sum \frac{(-1)^{j-1}}{2j-1} \cos\left[\frac{\pi}{2}(2j-1)\xi\right] \exp\left[-\left(\frac{\pi}{2}(2j-1)\right)^2 T\right]$

- It is not at all logical to aim at a solution of pore pressure as long as our practical aim is to obtain settlements (Janbu, 1965).
  - የምንፈልገው የዐፈሩን የዝብጠት መጠን እስከሆነ ድረስ መፍትሄውን በውኃ ግፊት አማካኝነት መፈለግ አጥጋቢ አይደልም።
- The classical theory does not take none-linearity of stress-strain behavior into account.
  - መደበኛው (የተርዛጊ) ንድፈ ሐሳብ የጫና /ውጥረትን ኢርቱዕ የሆነ ዝምድና ከግምት ውስጥ ያስገባም።
- For nonlinear stress-strain (which is naturally true for soils) the degree of consolidation has to be obtained from strain-depth distribution (Janbu, 1969).
  - የዐፈር ሜካኒካዊ ባሕርይ የሆነው ኢርቱዕ የሆነ የጫና ውጥረት ዝምድና ከውጥረት ጥልቀት ስርጭት (ዝምድና) መገኘት አለበት
- The total stress can be time dependent.
  - ሙሉው ጫና ጊዜ ጥገኛ ሊሆን ይችላል
- Consolidation may not be 1D.
  - የዐፈር ሜካኒካዊ ሥርዓት በአውታረ አንድ ሥርዓት ሙሉ በሙሉ ላይገለጥ ይችላል።

Janbu's strain based 1DC Theory

- Strain distn $t \rightarrow \infty$
  - ❑ $\varepsilon_1 = \varepsilon_s - \varepsilon_d(1 - \xi^r)$
- Basic strain based differential equation
  - ❑ $\frac{\partial \varepsilon}{\partial T} = \frac{\partial^2 \varepsilon}{\partial \xi^2} - r(r-1)\varepsilon_d \xi^{r-2}$
- Solution
  - ❑ $\varepsilon(t) = \varepsilon_1 - \sum C_N \cos(N\xi)\exp(-N^2 T)$
  - – $C_N = \frac{2\varepsilon_s}{N}\sin N - 2r\varepsilon_d \frac{\sin^{r+1} N}{N^{r+1}}$
  - – $N = \frac{\pi}{2}(2j - 1)$

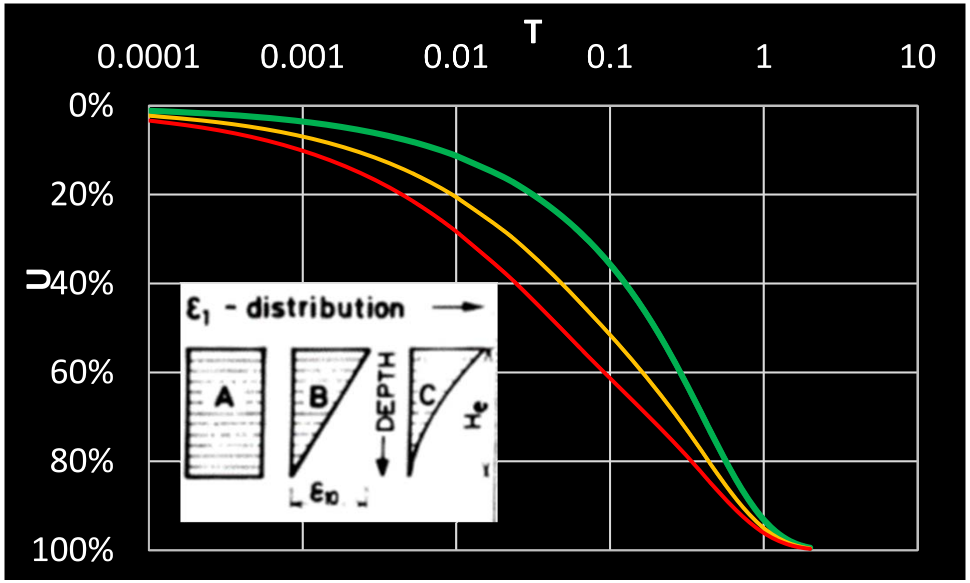


Janbu's…

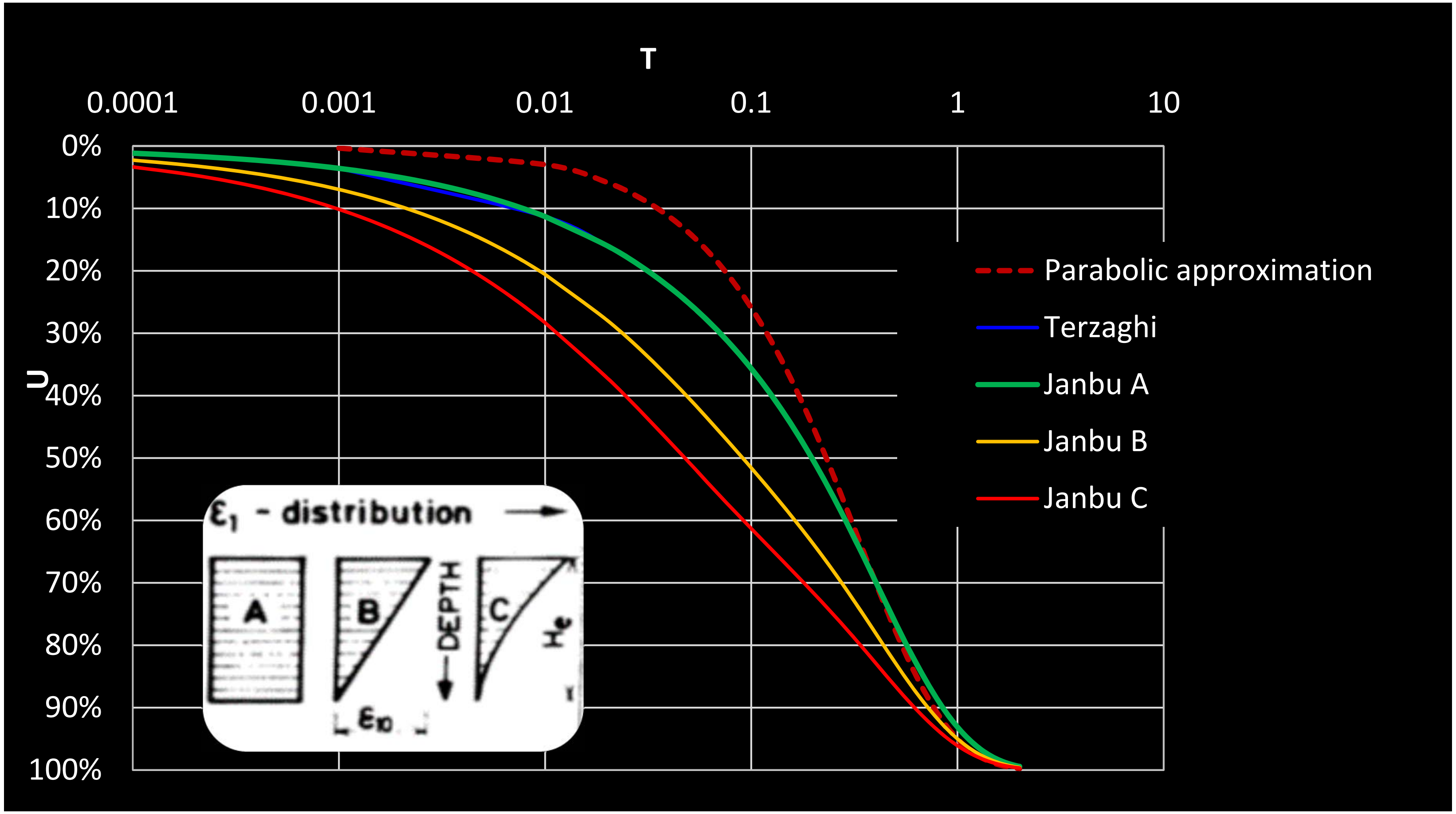
- Degree of consolidation
  - ❑ $U = \int_0^1 \varepsilon d\xi / \int_0^1 \varepsilon_1 d\xi = \frac{U_0 - f_s F(T)}{1 - f_s}$

where:

- $U_0 = 1 - 2\sum \frac{\sin^2 N}{N^2} \exp(-N^2 T)$
- $f_s = \frac{r\varepsilon_d}{(1+r)\varepsilon_s}$
- $F(T) = 1 - 2(r+1)\sum \frac{\sin^{2+r} N}{N^{2+r}} \exp(-N^2 T)$
- $N = \frac{\pi}{2}(2j - 1)$



Janbu’s…



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Parabolic approximation
Terzaghi
Janbu A
Janbu B
Janbu C
ε1 - distribution
A
B
C
DEPTH
He
ε10

Back to engineering
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- How do we use our knowledge of consolidation to solve practical problems?
- ስለስብሳቦሽ ያለንን ዕውቀት ተጠመን እንዴት ተግባራዊ የምሕንድስና መፍትሄዎችን ልንተገብር እንችላለን?

Applying preloading
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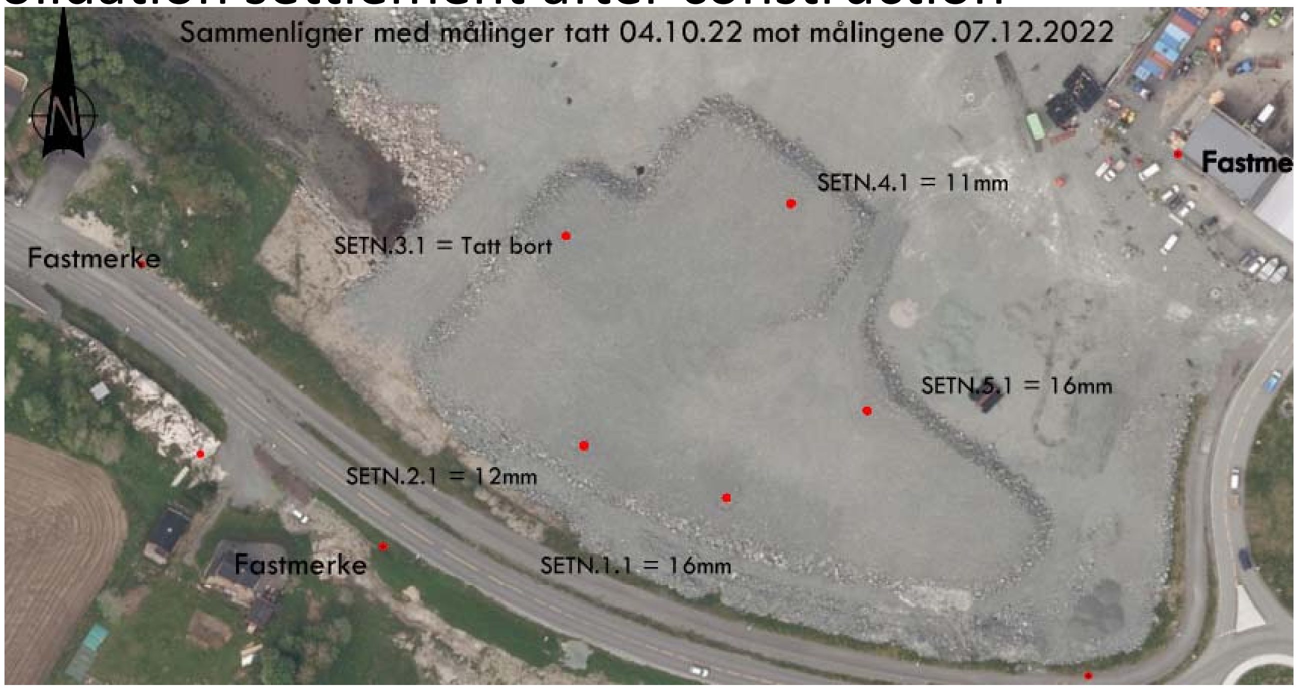
- Heaping a soil and let the soil under consolidate for sometime
- Remove the soil heap
- Implement the actual construction.
- This improves the bearing capacity of the underlying soil. It also reduces the consolidation settlement after construction



Applying drains
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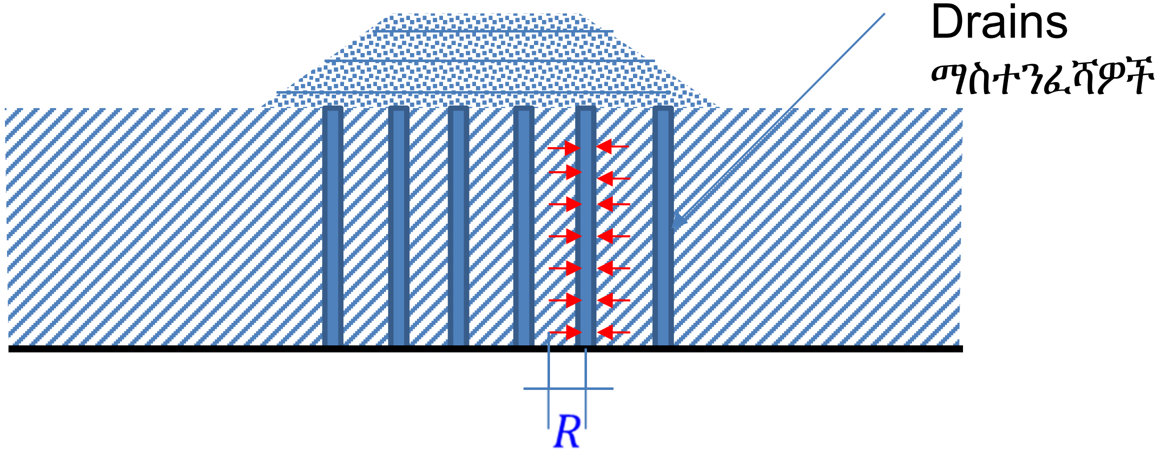
- Install drains to accelerate the consolidation process and improve the bearing strength of the soil.
  - $U \approx 1 - \exp\{-4T\}, T = \frac{c_r}{R^2} t$



Applying drains
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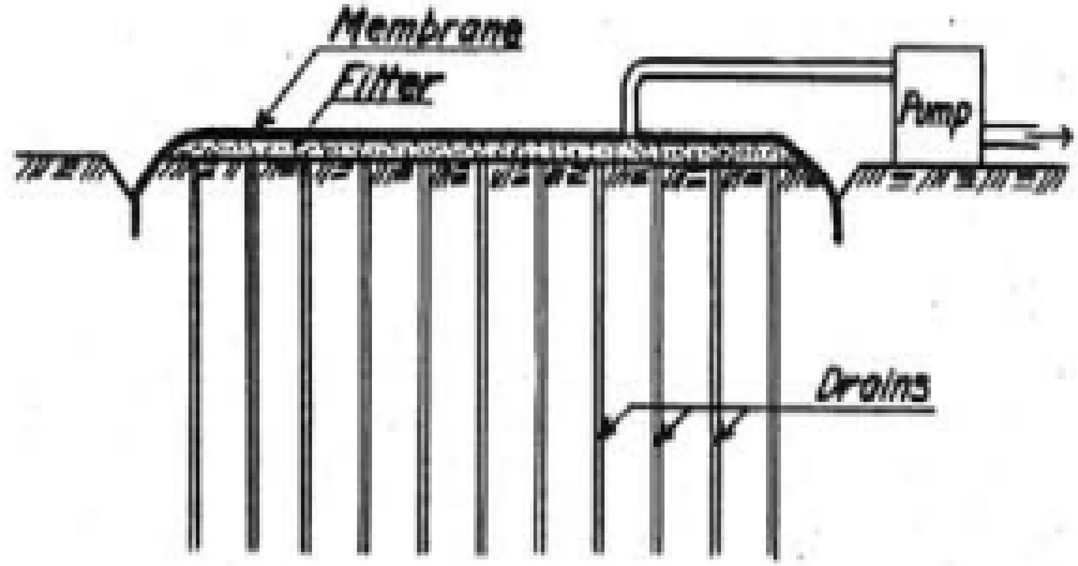
- Vacuum preloading to accelerate the consolidation process



Kjlleman (1952)

Asaoka's observational method

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Basic differential equation:

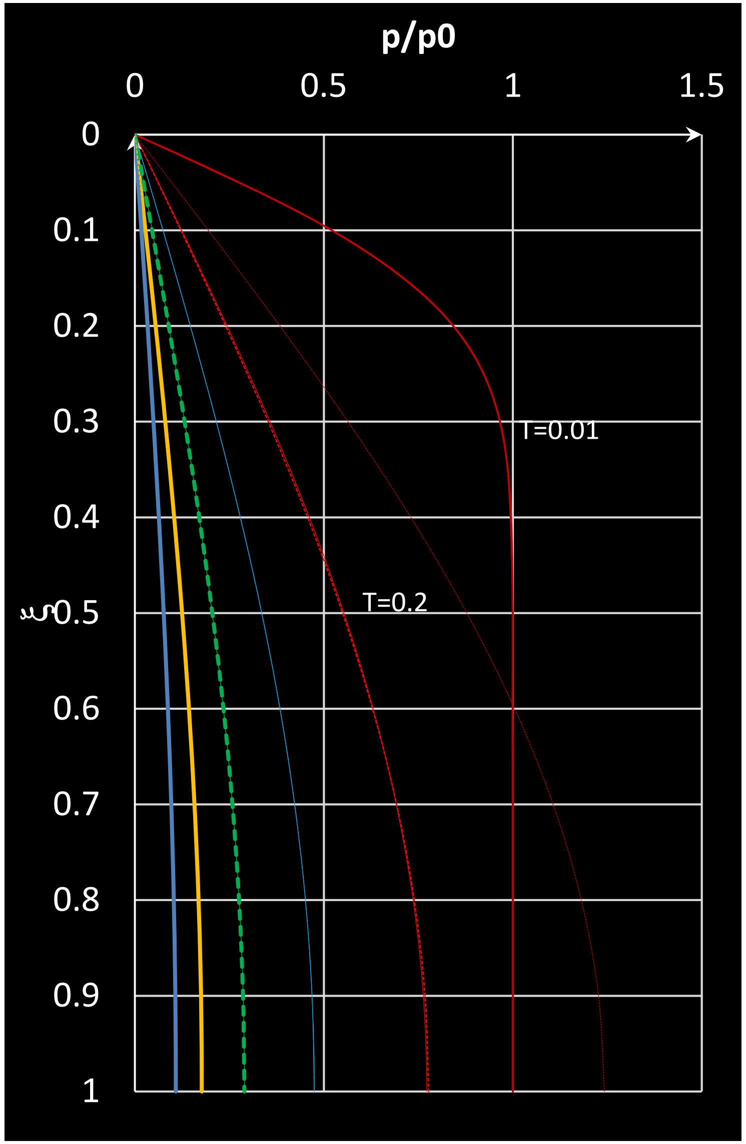
- ❑ $\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial z^2}$

The solution in terms of pore pressure

- ❑ $\frac{p}{p_0} = \frac{4}{\pi} \sum_{j=1}^{\infty} \frac{(-1)^{j-1}}{2j-1} \cos\left(\frac{(2j-1)\pi z}{2h}\right) \exp\left(-\frac{(2j-1)^2\pi}{4h^2}\right.$

For larger values of time, *i.e.*, for $\frac{ct}{h^2} > 0.2$ the higher order terms vanish, and the equation may be approximated by the first term:

- ❑ $\frac{p}{p_0} \approx \frac{4}{\pi} \cos\left(\frac{\pi z}{2h}\right) \exp\left(-\frac{\pi^2}{4h^2} c_v t\right)$



First order approximation
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The strain may then be obtained as:

- $\varepsilon = \frac{p_0}{M}\left(\frac{p}{p_0} - 1\right) = \frac{p_0}{M}\left(\frac{4}{\pi}\cos\left(\frac{(h-z)\pi}{2h}\right)\exp\left(-\frac{\pi^2}{4h^2}c_v t\right) - 1\right)$

The displacement is then found by integrating the strain over the depth:

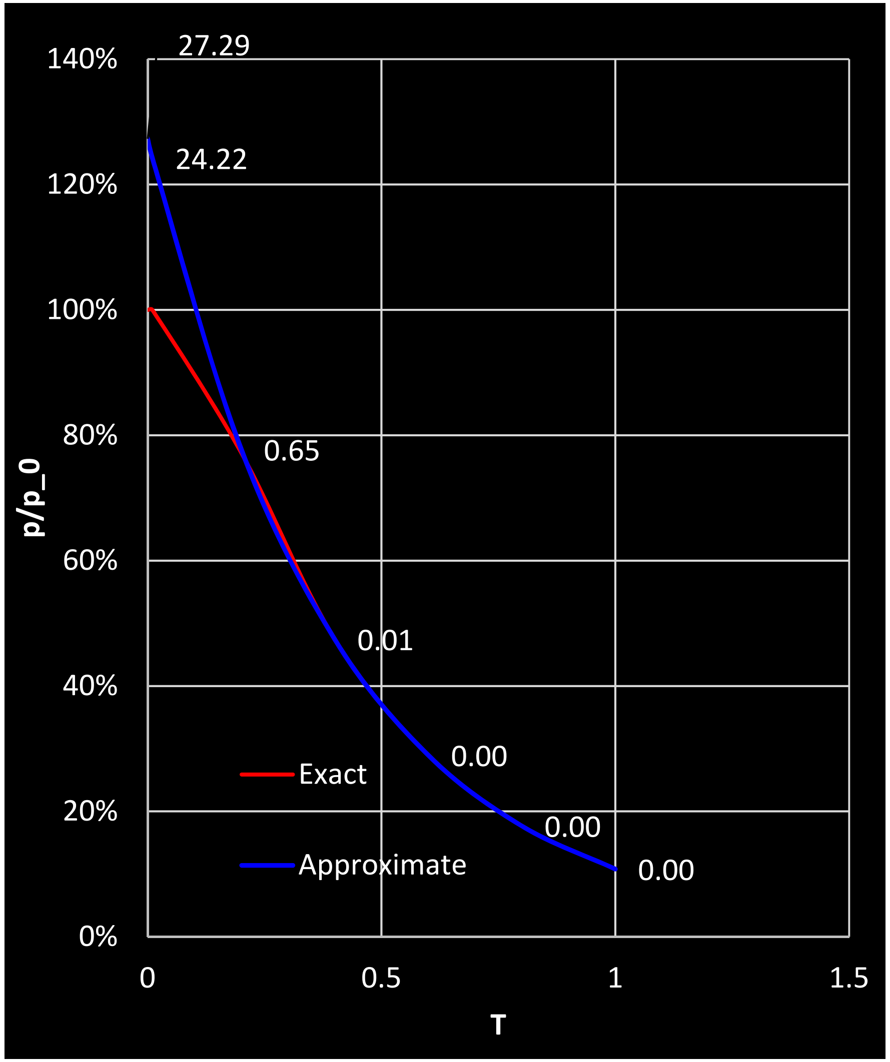
- $S = -\int_0^h \varepsilon dz = \frac{p_0}{M} h\left(1 - \frac{8}{\pi^2}\exp\left(-c_v\frac{\pi^2}{4h^2}t\right)\right)$

Simplifying:

- $\boldsymbol{S = S_\infty(1 - \delta\exp(-\beta t))}, \delta = \frac{8}{\pi^2}, \beta = c_v\frac{\pi^2}{4h^2}, S_\infty = \frac{p_0}{M}h$

First order autoregressive form (in which $b = \exp(-\beta\Delta t)$)

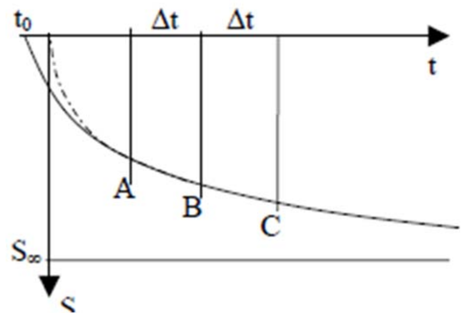
- $\boldsymbol{S_i = a + bS_{i-1}}, a = S_\infty(1 - b)$

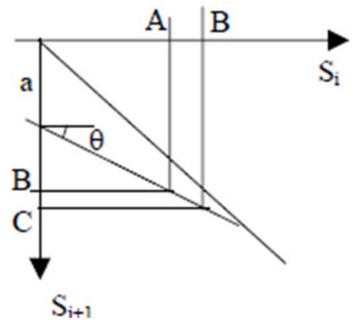


Asaoka's graphical method using first order approximation

- ❑ Let us have n+1 settlement observations, ($u_0, u_1, \ldots, u_n$) generated by a constant external load.
- ❑ Using these observations plot n points $(u_k, u_{k-1})$for k=1,2, …, n on the $\left(u_j, u_{j-1}\right)$ coordinate system
- ❑ The plot will be linear and tends towards the end of consolidation and therefore enables us to predict future settlements (note that the (u_k, u_{k-1}) plot for the end of will have a slope of 45 degrees for the end of consolidation.)
- ❑ From the plot of observation points (u_k, u_{k-1}) one finds a and θ

- Basis (first order approximation):
  - ❑ $S_i = a + bS_{i-1}$
- From the plot:
  - ❑ $S_\infty = \frac{a}{1-b}$, $b = \tan\theta$
  - ❑ $\frac{c_v}{H^2} = \frac{4(1-b)}{\pi^2 \Delta t}$





Asaoka-Reliability – In a Bayesian framework

The aim of this part is to adopt the Bayesian framework for the prediction of the parameters a and b =tanθ conditioned on measurements. Asaoka claims that the method has an advantage in that "the predicted value is given with its reliability"

$$S(t_{i+1}) = a + b \cdot S(t_i) + \epsilon_i$$

where ϵ_i is a random noise. Asaoka proposes ϵ_i to be Gaussian distributed with a 0-mean value.

The usual procedure is as follows:

- ❑ Assume prior distributions (Asaoka suggested using a Gaussian distribution.) We could also assume non-informative priors such as uniform distributions, say P(a), P(b), and P(σ) if we wish to start from "I don't know."

- ❑ Define a likelihood function: $P(S(t_{i+1})|a,b,S(t_i),\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{\left((t_{i+1})-(a+b\cdot S(t_i))\right)^2}{2\sigma^2}\right)$, where σ^2 is the variance of the measurement errors. The joint likelihood for all observations is the product of the likelihoods for individual data points, i.e., $L(a,b,\sigma|Data) = \prod_{i=1}^{n-1} P(S(t_{i+1})|a,b,S(t_i),\sigma)$
- ❑ Find the posterior distributions for a, b and σ using Baye's theorem: $P(a,b,\sigma|Data) \propto L(a,b,\sigma|Data) \cdot P(a) \cdot P(b) \cdot P(\sigma)$. The posterior distribution combines the prior beliefs about a and b with evidence provided by the observed data to give updated estimates of these parameters.

- ❑ Predict the ultimate settlement: $S_\infty^{(j)} = \frac{a^{(j)}}{1-b^{(j)}}$

Recap

- Elementary mechanical behaviour of soils
- Generation and dissipation of pore pressure
- Engineering problems related to generation and dissipation of pore pressures
- Theory of consolidation
- Back to engineering… how to use knowledge of consolidation to solve engineering problems related to generation and dissipation of pore pressures
- Asaoka’s observational method

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